Flight-to-Quality via the Repo Market*

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Abstract

The European debt crisis of 2011 has been characterized by an unprecedented divergence in borrowing costs for euro area members. While 'peripheral' government bond yields increased to unprecedented levels, yields on German and other 'core' bonds strongly declined, even though their CDS-spreads reached an all-time high in 2011. To reconcile this flight-to-quality, I propose a model of a financially integrated monetary union in which heterogeneous sovereign borrowers issue bonds subject to default risk. Investors value the collateral service of government bonds, which decreases in haircuts that are specified by the central bank in its collateral framework. In a union-wide fiscal crisis, larger haircuts increase the yields of riskier government bonds and also imply a contraction of aggregate collateral supply. This makes the collateral service of the safest available bonds more valuable to investors: yields on the safest bonds decline, even though their default risk increases. Using the calibrated model, I show that a full collateral backstop policy accepting all bonds with zero haircuts *during a fiscal crisis* reduces the dispersion of government bond spreads and reduces sovereign risk in the monetary union. This result lends support to the ECB's decision to temporarily suspend minimum rating requirements on government bonds in response to the Covid-19 shock.

Keywords: Sovereign Risk, Flight-to-Quality, Collateral Premia, Collateral Policy, Lender-of-last-resort, Investor-of-last-resort

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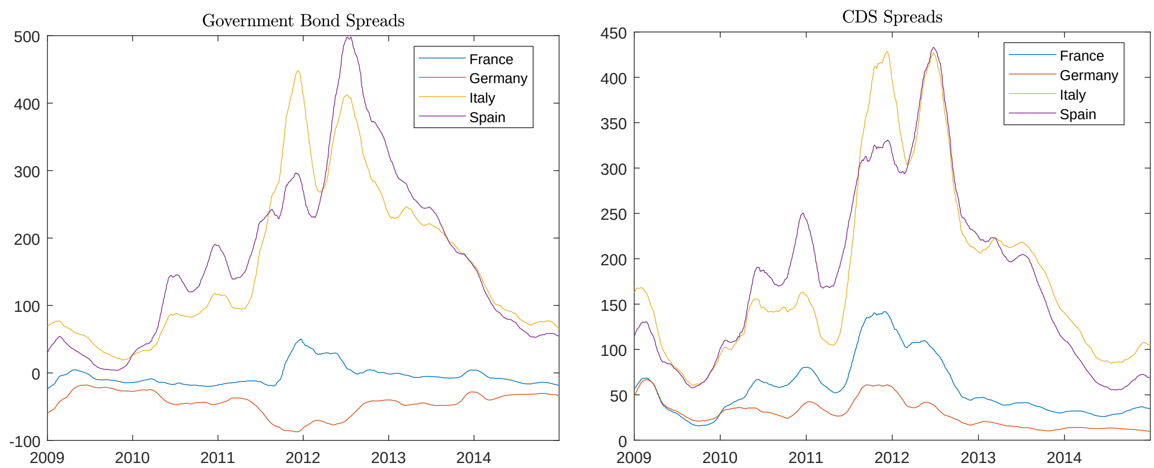
1 Introduction

The European debt crisis of 2011 has been characterized by an unprecedented divergence in borrowing costs for euro area members. The left panel of Figure 1 depicts government bond spreads - computed over default risk-free interest rate swaps - around the European debt crisis for the four largest sovereign borrowers. While the bond spreads of France, Italy, and Spain exhibit substantial increases relative to their pre-crisis level, German bond spreads declined to around -100bp. At the same time, credit default swaps (CDS) show a positive co-movement, as shown in the right panel of Figure 1: similar to French, Italian, and Spanish CDS-spreads, the German CDS-spread also strongly increased in the fall of 2011, peaking at almost 80pb. This suggests that investors (1) required a higher default risk compensation on *all* euro area government bonds, including the German bund, but (2) were still willing to pay lower yields to hold the German bunds.

Figure 1: Government bond- and CDS-spreads, 5 year maturity, rolling means



Notes: Spreads are computed with respect to interest rate swaps of corresponding maturity. All values are 60 business day rolling means and expressed in basis points. Source: *Thomson Reuters Datastream.*

To reconcile this puzzling observation, this paper proposes a model of a monetary union with heterogeneous governments (countries) receiving random tax revenues and issuing long-term bonds to competitive investors. There are five key assumptions. *First*, governments issue bonds subject to default risk, which implies that CDS-spreads for all countries are strictly positive.[1] *Second*, there is no fiscal coordination among union members: country-specific shocks to fiscal revenues are not optimally shared among union members and default risk is heterogeneously distributed across countries. *Third*, investors use government bonds to collateralize repos, either on the interbank market or with the central bank: government bond spreads contain a default risk component and a collateral premium, such that CDS-spreads are higher than corresponding bond spreads for each country.[2] *Fourth*, the monetary union is financially integrated in the sense that investors can use all government bonds as collateral equally well. Collateral premia therefore depend negatively on *aggregate collateral supply*, which will be defined as the haircut-adjusted market value of all government bonds outstanding. *Fifth*, the collateral premium on each government bond declines in its default risk, since private repo market participants and the central bank specify haircuts that negatively depend on sovereign credit ratings, see Bindseil and Papadia (2006), Nyborg (2017), and Orphanides (2017) for a discussion of central bank collateral frameworks and Nguyen (2020), Fontana and Scheicher (2016), and Jiang et al. (2021) for empirical evidence.

The cross-sectional distribution of sovereign risk, government bond spreads, and haircuts is jointly determined with aggregate collateral supply in equilibrium. An unanticipated negative *aggregate* shock to tax revenues increases sovereign default risk and CDS-spreads for all countries, consistent with the data.[3] Elevated sovereign default risk is also associated with rating downgrades. Holding central bank collateral policy constant, this mechanically increases haircuts on all government bonds: aggregate collateral supply contracts and collateral service becomes more valuable to investors. To see how such a contraction of aggregate collateral supply affects the cross-section of government bond

[1]Default risk on local currency debt is relevant in a monetary union, since governments can not use currency devaluation to decrease the real value of their debt service burden if monetary policy is delegated to a supra-national central bank.

[2]At the peak of the European debt crisis, government bond spreads exceeded CDS-spreads for the riskiest countries. This has been attributed to market illiquidity, which does not affect CDS-spreads, but increases government bond spreads, see Passadore and Xu (2020). The interaction of default risk and collateral premia is robust to endogenizing market illiquidity as shown in Kaldorf and Roettger (2021). Therefore, this layer of complexity is omitted in this paper.

[3]The relevance of aggregate fiscal risk is supported empirically by Monfort and Renne (2013).

spreads, it is helpful to decompose the change of collateral premia into a *collateral valuation* effect, reflecting the value that an additional unit of collateral provides to investors, and a *haircut* effect, which captures how much collateral service any specific government bond provides.

The haircut effect dominates for countries most affected by default risk, who exhibit a decline of collateral premia. Their government bond spreads increase due to higher default risk compensation *and* due to lower collateral premia. At the same time, the collateral valuation component dominates for countries that are only weakly affected by default risk and, therefore, subject to smaller haircuts. Even though their haircuts increase during a fiscal crisis, the collateral premium on their bonds increases, which reduces their government bond spreads. In a calibration to euro area data, I demonstrate that the model can quantitatively replicate the dynamics of haircuts, CDS-spreads and government bond spreads observed during the European debt crisis. Simulating a panel of government bonds, I furthermore show that the model also performs reasonably well in reconciling the (untargeted) cross-sectional evidence on the effect of country-specific fiscal conditions on the non-pecuniary benefits of their government bonds, which is provided for euro area data by Jiang et al. (2021).

Using the calibrated model, I evaluate to which extent the ECB's collateral framework can be used to close the gap between yields on 'peripheral' and 'core' bonds during a fiscal crisis.[4] In addition to maintaining a sufficiently high collateral supply - which ensures smooth functioning of financial markets and monetary policy implementation - containing centrifugal forces threatening the currency union has been a major challenge for policymakers. This has been motivated on several different grounds: large spread heterogeneity can be costly at the monetary union level through contagion effects (Morelli et al., 2019), joint strategic default (Arellano et al., 2017), or direct spillover costs of

[4]Notably, the ECB engaged in collateral easing policies prior to announcing large-scale central bank interventions on the government bond market. However, as demonstrated in D'Amico et al. (2018) for the US and Schlepper et al. (2018), Arrata et al. (2020), and Corradin and Maddaloni (2020) for the euro area, quantitative easing policies even increase high collateral valuation effects by taking government bonds out of the market, thereby potentially amplifying the flight-to-quality effect studied in this paper.

sovereign default to other member states (Tirole, 2015).

In a fiscal crisis, the central bank can directly affect the cross-section of government bond yields through a *temporary* relaxation of collateral policy. Decreasing haircuts on all countries has a direct positive effect on collateral premia and reduces bond yields for all countries, ceteris paribus. At the same time, this relaxation increases aggregate collateral supply, making the collateral service of the safest bonds less valuable, such that their yields increase. Specifically, a *full collateral backstop* policy that accepts all bonds as collateral without haircuts most effectively reduces the cross-sectional dispersion of government bond spreads during a fiscal crisis.[5] This reduces overall sovereign risk in the monetary union during the crisis period, since it makes debt rollover easier for high-risk borrowers. Taken together, my results lend support to the ECB's temporary suspension of minimum rating requirements in April 2020 in response to the Covid-19 shock and the associated disruptions on financial markets.[6]

The full collateral backstop result is related to canonical lender-of-last-resort (LOLR) policy. Following the survey article by Freixas et al. (2000), central banks in their role as LOLR ensure that all investors with 'good collateral' are able to borrow from the central bank whenever alternative funding sources via the private sector are not available. The design of such policies trades off the benefits of LOLR-interventions, for example avoiding bank failure, contagion effects, and contractions in real sector lending, against its costs, for example due to moral hazard in the banking system and by fiscal policy.

When applied to the single-country case, where government bonds are issued in domestic currency and therefore typically are *nominally risk-free*, the LOLR-literature usually takes a sufficient aggregate supply of 'good collateral' for granted. However, canonical LOLR-policy might be infeasible in a monetary union during a fiscal crisis, because of an insufficient supply of 'good collateral'. LOLR-policy therefore has to ensure that fundamentally solvent, but illiquid investors are able to tap liquidity facilities, even if

[5]Throughout the paper, I use the term *collateral backstop* to make explicit the distinction to an *outright backstop* policy that buys a large amount of distressed government bonds on secondary markets, which is discussed in Corsetti and Dedola (2016).

[6]The ECB press release can be accessed via this link.

the collateral value of government bond declines. The central bank acts similar to an *investor*-of-last-resort (Buiter and Rahbari, 2012). However, in contrast to using outright government bond purchases to reduce heterogeneity of government borrowing costs (Canofari et al., 2018), which introduces a non-trivial risk management consideration to the central bank's policy problem (Caballero et al., 2020), a full collateral backstop does not transfer risk to the central bank balance sheet, since collateral frameworks apply to central bank *repurchase agreements*. Therefore, a full collateral backstop is partially successful at carrying out investor- and lender-of-last-resort policies simultaneously.

It should be noted that, throughout my analysis, I maintain the assumption that both the aggregate fiscal shock and the full collateral backstop are unanticipated. This effectively eliminates moral hazard and time-consistency issues in both the governments' and the central bank's policy problem. The effects of government bond haircuts on fiscal policy under sovereign are discussed in Kaldorf and Roettger (2021), but can reasonably assumed to be less relevant during a flight-to-quality episode.

Related Literature I draw on three distinct strands of literature. The collateral channel around which this paper is centered relates to a group of papers studying the interactions of non-pecuniary benefits and sovereign risk in advanced economies. The negative effect of non-pecuniary benefits on government bonds yields has been documented in Jiang et al. (2021). This divergence in borrowing costs during a fiscal crisis negatively affecting all euro area members has been associated with a 'flight-to-liquidity' effect by De Santis (2014). I contribute to this strand of literature by providing an equilibrium characterization that can be used to study counterfactual policies. My model also builds on Bolton and Jeanne (2011), who study convenience yield in a setting with high and low risk countries borrowing from a representative investor under perfect financial integration. Luque (2021) proposes a repo market model with heterogeneous banks using low-risk and high-risk sovereign borrowers as collateral and shows that flight-to-liquidity can arise if high-risk bonds lose collateral eligibility. Auray et al. (2018) propose a two-country model with an interbank market that accounts for sovereign-bank doom loops but abstracts from collateral premia. Their model also shows that unconventional central bank interventions can improve welfare during a crisis. All three papers assume that low risk borrowers issue bonds free of default risk, which is at odds with the high German CDS-spread observed in 2011.

From a methodological perspective, I relate to a group of papers studying sovereign default in monetary union, see for example Arellano et al. (2017), Costain et al. (2021), and De Ferra and Romei (2021). In contrast to these papers, rather than having two borrower types ('core' and 'periphery'), I consider a *continuum of borrower types* following Le Grand and Ragot (2021). Different to their setup, I follow Jiang et al. (2021) in assuming that governments trade with investors rather than among each other. This assumption facilitates a straightforward decomposition of government bond spreads into default risk compensation and collateral premia and also eliminates the multiplicity around which the discussion in Le Grand and Ragot (2021) is centered.

From a policy perspective, the central bank's role as investor/lender-of-last-resort in the presence of sovereign risk has also received considerable attention in recent years. Corsetti and Dedola (2016) and Reis (2017) focus on unconventional central bank policies, such as outright purchases of distressed government bonds. This literature typically operates in a multiple-equilibria setting (see also Lorenzoni and Werning, 2019), where LOLR policies help to coordinate on the low-debt and low-default equilibrium with higher welfare. Closest to my approach are Bocola and Dovis (2019), who propose a single-borrower model and show that LOLR *announcements* - exemplified by Mario Draghi's 'Whatever it takes'-speech in summer 2012 - are an effective tool to reduce sovereign risk by peripheral euro area borrowers. My paper complements these approaches from a conceptual perspective, since the equilibrium is unique in my model, and from a policy perspective, since collateral policy is typically considered a conventional monetary policy instrument.

2 Model

Time is discrete and denoted by $t = 1, 2, ...$ and there is no aggregate risk. Each period corresponds to one year. There are two risk-neutral groups of agents, investors and governments, trading with each other on the government bond market.

Investors Investors discount the future at constant rate r^{rf}. They buy government bonds b^j_{t+1} as investment object and in addition value the collateral service provided by government bond holdings. To introduce a willingness to pay collateral premia, I assume that investors directly draw utility from holding eligible collateral.[7] Collateral benefits are represented by a CARA-function

$$\mathcal{L}(\overline{B}) = -\frac{l_0}{l_1} \exp\left\{-l_1 \overline{B}\right\} \tag{1}$$

with CARA-coefficient l_1 and weighting parameter l_0. The economy exhibits perfect financial integration in the sense that investors can use all government bonds equally well as collateral if they are subject to the same central bank haircut. While the substitutability of collateral service is difficult to test empirically, Delatte et al. (2016) and Luque (2021) provide evidence for collateral-driven portfolio reallocations in response to rating downgrades or increased margin requirements for risky (peripheral) government bonds. Armakolla et al. (2019) empirically document that higher haircuts on government bond issued by a specific country are associated with a decrease in the usage of these bonds as collateral, suggesting that investors are at least to a certain degree able to substitute between different securities that are eligible as collateral. Therefore, collateral benefits

[7] This is similar to the literature on convenience yield, for example Krishnamurthy and Vissing-Jorgensen (2012) and Greenwood et al. (2015). An observationally equivalent formulation can be obtained by assuming that investors need to settle uninsurable liquidity shocks by borrowing on the interbank repo market or from the central bank against eligible collateral.

(1) depend on aggregate collateral supply $\overline{B}$, which is defined as

$$\overline{B} \equiv \int_j \frac{(1-\kappa_t^j)k_t^j}{1+r^{rf}} b_{t+1}^j dj \ . \tag{2}$$

Collateral supply consists of three distinct parts: bond quantities b_{t+1}^j, discounted expected payoffs $\frac{k_t^j}{1+r^{rf}}$, and haircuts κ_t^j. The expected payoff from purchasing a bond issued by government j and the haircut applied to this bond in period t are linked to the debt and default decision in period $t+1$, which will be described below. It is natural to interpret $\frac{k_t^j}{1+r^{rf}}$ as the pledgeable value of bond j: in the (unmodeled) event of an investor default between period t and $t+1$, the central bank seizes the bond and is entitled to its payoff (Bindseil and Papadia, 2006). To cushion against adverse price movements of the pledged collateral, the central bank additionally applies haircuts κ_t^j to government bonds.[8]

The benefit of holding one additional unit of collateral is then given by the first derivative of (1), i.e. by $l_0 \cdot e^{-l_1 \overline{B}}$. Following Bindseil (2014), default risk is mapped into haircuts using the simple functional form

$$\kappa_t^j = \min\left\{ \left(F_t^j\right)^\eta, \overline{\kappa} \right\} \ . \tag{3}$$

The parameter η governs the sensitivity of haircuts to default risk F_t^j, which is interpreted as a variable beyond central bank control. This is motivated by the heavy usage of government bonds as collateral on the private repo market, where haircuts are either negotiated bilaterally or set by a central clearing counterparty. During the European debt crisis, peripheral government exhibited large haircut increases (Gabor and Ban, 2015) on private markets. In contrast, $\overline{\kappa}$ is the central bank policy parameter of interest and defines a maximum haircut which applies to all bonds, irrespective of their default

[8] If the central bank lends funds l_t at the risk-free rate against a risky bond j, full collateralization in the absence of collateral default risk requires $l_t(1+r^{rf}) = (1-\kappa_t^j)k_t^j b_t^j$. The maximum loan size a bank can obtain by pledging this bond is therefore given by $\frac{(1-\kappa_t^j)k_t^j}{1+r^{rf}} b_{t+1}^j$.

risk. This effective cap is consistent with the ECB's practice already prior to the financial crisis (Buiter and Sibert, 2005) and during the European debt crisis (Nyborg, 2017). Since central bank haircuts apply to standing facilities, investors are able to obtain $\frac{1-\kappa}{1+r^{rf}}$ units of funding even for the riskiest government bonds in the monetary union. This reflects the *collateral backstop* notion of central bank collateral frameworks, with $\overline{\kappa} = 0$ corresponding to the full backstop. Solving the maximization problem of investors delivers a pricing condition for government bond j

$$q_t^j = \frac{1 + (1 - \kappa_t^j) \cdot l_0 \cdot e^{-l_1 \overline{B}}}{1 + r^{rf}} \cdot k_t^j \; . \tag{4}$$

The bond pricing condition contains two parts: the discounted expected payoff $\frac{k_t^j}{1+r^{rf}}$ and a collateral premium $(1 - \kappa_t^j) \cdot l_0 \cdot e^{-l_1 \overline{B}}$, which in turn can be decomposed into a *collateral valuation* component $l_0 \cdot e^{-l_1 \overline{B}}$ and the term $1 - \kappa_t^j$, to which I will also refer as the *haircut* component.

Governments There is a mass-one continuum of governments that supply a domestic public good g_t^j. Each government receives random tax revenues y_t^j, which follow a Markov chain on a discrete grid of revenue realizations with transition matrix Π_y. The cdf associated with the distribution of y_{t+1}^j, conditional on current revenues y_t^j will be denoted by $F(y_{t+1}^j | y_t^j)$.

Governments are only able to use a part of their revenues $\overline{\Theta}\theta_t^j < 1$ for the repayment of debt obligations. This can be interpreted as the political ability to use public revenues for debt service. Its cross-country average is determined by the parameter $\overline{\Theta} < 1$. Since the political ability to use public funds for debt service can fluctuate over time, I add the stochastic component θ_t^j which has mean one and follows a two-state Markov chain with transition matrix Π_θ. Its cdf will be denoted by $F(\theta_{t+1}^j | \theta_t^j)$. The time-varying willingness to repay is set such that countries in the low state $\theta_t^{risky} < \theta_t^{safe}$ will have a higher default risk, ceteris paribus. In the context of the euro area debt crisis, countries with θ_t^{safe} can be interpreted as the 'core', while those drawing the low realization θ_t^{risky} represent the 'periphery'.

Governments issue bonds to smooth consumption across time and states. The payoff structure of government bonds follows Chatterjee and Eyigungor (2012). Bonds pay a fixed coupon c and mature with probability $0 < \pi \leq 1$ each period, while the non-maturing share is rolled over at current market price q_t^j. Following the ability-to-repay approach, the government repays its bonds if transferable government revenues $\overline{\Theta}\theta_t^j y_t^j$ fall short of current repayment obligations $(\pi + c)b_t^j$. The expected payoff k_t^j is thus given by

$$k_t^j = (\pi + c)\Big\{1 - F\Big(\frac{(\pi + c)b_{t+1}^j}{\overline{\Theta}}\Big|\theta_t^j, y_t^j\Big)\Big\} + (1 - \pi)\mathbb{E}_t\left[q_{t+1}^j\right] .$$

Here, $F\Big(\frac{(\pi+c)b_{t+1}^j}{\overline{\Theta}}\Big|\theta_t^j, y_t^j\Big)$ is the default probability, which can be expressed using the joint cdf of next period's exogenous state $(\theta_{t+1}^j, y_{t+1}^j)$, conditional on the current exogenous state (θ_t^j, y_t^j). Plugging this into the bond pricing condition (4) yields the bond price *schedule*, which depends on the exogenous state (θ_t^j, y_t^j) and government j's choice variable b_{t+1}^j:

$$q(b_{t+1}^j, \theta_t^j, y_t^j) = \frac{1 + \left(1 - \kappa(b_{t+1}^j, \theta_t^j, y_t^j)\right) \cdot l_0 \cdot e^{-l_1\overline{B}}}{1 + r^{rf}} \times \left\{(\pi + c)\left(1 - F\Big(\frac{(\pi + c)b_{t+1}^j}{\overline{\Theta}}\Big|\theta_t^j, y_t^j\Big)\right) + (1 - \pi)\mathbb{E}_t\left[q\Big(\mathcal{B}(b_{t+1}^j, \theta_{t+1}^j, y_{t+1}^j), \theta_{t+1}^j, y_{t+1}^j\Big)\right]\right\} . \quad (5)$$

The bond price schedule links the expected pecuniary payoff to the valuation of collateral service provided by government bond j. The expected payoff consists of the expected coupon payment c and redemption share π in period $t+1$ as well as the expected rollover value of bonds $(1 - \pi)\mathbb{E}_t[q(\cdot)]$. Note that the rollover value depends on the debt policy $\mathcal{B}(b_{t+1}^j, \theta_{t+1}^j, y_{t+1}^j)$ in $t + 1$, which in turn depends on current debt issuance b_{t+1}^j and the shock realizations in $t + 1$. Collateral service is negatively related to default risk via the haircut function $\kappa(b_{t+1}^j, \theta_t^j, y_t^j)$, while collateral valuation depends on aggregate collateral

$\overline{B}$. Taken as given investors' bond price schedule, each government j maximizes

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u(g_t^j)\right] \quad \text{with} \quad u(g_t^j) = \begin{cases} g_t^j - \underline{g} & \text{if } g_t^j > \underline{g} \\ -\infty & \text{else} \end{cases} \tag{6}$$

subject to the sequence of budget constraints

$$\begin{aligned} g_t^j = (1-\overline{\Theta}\theta_t^j)y_t^j + \mathbb{1}\{\overline{\Theta}\theta_t^j y_t^j > (\pi+c)b_{t+1}^j\}\left[\overline{\Theta}\theta_t^j y_t^j - (\pi+c)b_t^j\right] \\ + q(b_{t+1}^j, \theta_t^j, y_t^j)\left(b_{t+1}^j - (1-\pi)\,b_t^j\right) \ . \end{aligned} \tag{7}$$

In case of default, the government loses the transferable share $\overline{\Theta}\theta_t^j y_t^j$, but does neither redeem the maturing share π of bonds nor services the coupon payment c. Since there are no delays in restructuring, the government is always able to issue bonds b_{t+1}^j and roll-over the non-maturing share of its bonds outstanding $(1-\pi)b_t^j$. The maximization problem for each government j can be written recursively as

$$\mathcal{V}(b_t^j, \theta_t^j, y_t^j) = \max_{b_{t+1}^j} \ u(g_t^j) + \beta\mathbb{E}\left[\mathcal{V}(b_{t+1}^j, \theta_{t+1}^j, y_{t+1}^j)\right] \quad \text{subject to (7)} \ . \tag{8}$$

Solving the maximization problem yields the debt policy $\mathcal{B}(b_t^j, \theta_t^j, y_t^j)$, which is not available in closed form. Evaluating the bond price schedule (5) at the debt policy then delivers the equilibrium bond price

$$\mathcal{Q}(b_t^j, \theta_t^j, y_t^j) = q\left(\mathcal{B}(b_t^j, \theta_t^j, y_t^j), \theta_t^j, y_t^j\right) \ . \tag{9}$$

Equilibrium The recursive competitive equilibrium of the model is given by the bond price schedule $q(b_{t+1}^j, \theta_t^j, y_t^j)$, equilibrium bond price $\mathcal{Q}(b_t^j, \theta_t^j, y_t^j)$, debt policy $\mathcal{B}(b_t^j, \theta_t^j, y_t^j)$, value function $\mathcal{V}(b_t^j, \theta_t^j, y_t^j)$, and the cross-sectional country distribution $G(b_t^j, \theta_t^j, y_t^j)$, such that

- The debt policy solves the government problem (6) and the value function satisfies the Bellman equation (8).

- Bond price schedule and the equilibrium bond price satisfy (5) and (9).
- Aggregate collateral supply $\overline{B}$ is consistent with the country distribution over idiosyncratic states.

Credit Default Swaps The model permits the pricing of a security with a payoff-structure similar to a CDS written on government bond j.[9] The pricing schedule for CDS is given by the recursion

$$q^{CDS}(b^j_{t+1}, \theta^j_t, y^j_t) = \frac{1}{1+r^{rf}} \times \left\{ \left(1 - F\left(\frac{(\pi+c)b^j_{t+1}}{\overline{\Theta}} \Big| \theta^j_t, y^j_t\right)\right)(\pi+c) + (1-\pi)\mathbb{E}_t\left[q^{CDS}\left(\mathcal{B}(b^j_{t+1}, \theta^j_{t+1}, y^j_{t+1}), \theta^j_{t+1}, y^j_{t+1}\right)\right]\right\}. \tag{10}$$

Note that the continuation value is evaluated at the equilibrium debt policy. In contrast to the bond pricing condition (4), a CDS-type security only reflects fundamentals, i.e., default risk implied by the government's debt choices and does not contain a collateral valuation component.

3 Quantitative Analysis

To evaluate the model's quantitative properties, this section presents a calibration to euro area data and shows that it can replicate a flight-to-quality episode that is consistent with the cross-sectional dynamics of haircuts, government bond spreads, and CDS-spreads during the European debt crisis. Using the calibrated model, I demonstrate how central bank collateral policy can reduce the cross-sectional dispersion of government bond spreads and the overall level of CDS-spreads during a fiscal crisis.

[9] In the real world, the payoff of this security is replicated by a long position in the risk-free asset and a short position in a CDS.

3.1 Calibration

The model is solved using value function iteration on a discrete grid for debt b_t^j and the exogenous idiosyncratic states θ_t^j and y_t^j. Details on the computational algorithm and its parameters are deferred to Appendix B. Using the stationary competitive equilibrium, I introduce a fiscal crisis by subjecting the economy to an unanticipated shock to the average share of transferable government revenues $\overline{\Theta}$. Specifically, I assume that $\overline{\Theta}^{crisis} < \overline{\Theta}$, which makes default more likely for any given debt choice b_{t+1}^j and any exogenous state (θ_t^j, y_t^j). The aggregate shock is assumed to last for one period, after which the economy reverts back to the stationary equilibrium with certainty. Both exogenous idiosyncratic state variables are assumed to follow an AR(1)-process in logs

$$\log(\theta_t^j) = \rho_\theta \log(\theta_{t-1}^j) + \sigma_\theta \nu_t^j \ , \quad \nu_t^j \sim N(0,1) \ , \tag{11}$$

$$\log(y_t^j) = \rho_y \log(y_{t-1}^j) + \sigma_y \epsilon_t^j \ , \quad \epsilon_t^j \sim N(0,1) \ , \tag{12}$$

which will be discretized using the method proposed by Tauchen (1986), where (ρ_y, σ_y) can be estimated directly from tax revenues. The data for all countries that were members of the euro area in 2008 are obtained from the *St Louis Fed* database. I estimate (12) separately for each country from 1995 to 2019 and take the median estimate $\rho_y = 0.76$ and $\sigma_y^2 = 0.0032$. The maturity parameter $\pi = 0.2$ implies an average maturity of 5 years, which is an important benchmark maturity for euro-denominated government bonds and a typical value used in the literature. Setting $c = 0.045$ reflects the average coupon rate on Italian government bonds, as in Kaldorf and Roettger (2021). The sensitivity of haircuts to default risk is normalized to $\eta = 0.5$, while the haircut cap $\overline{\kappa} = 0.4$ is set to the average extraordinary ECB haircut applied to Greek and Cypriot government bonds during the euro area debt crisis (Nyborg, 2017).

The remaining eight parameters $\{\beta, \underline{g}, \rho_\theta, \sigma_\theta, \overline{\theta}, \overline{\theta}^{crisis}, l_0, l_1, \}$ are jointly calibrated to match several characteristics of the cross-sectional distribution over debt-to-GDP ratios, government bond spreads, CDS-spreads, and haircuts. Specifically, I use the upper and lower quartile (corresponding to Germany and Italy in the data) in normal times and during a fiscal crisis. Pre-crisis spreads and haircuts are based on the sample from 2009-01-01 to 2011-06-30. I use the sample period associated with the European debt crisis from 2011-07-01 to 2012-06-30 to represent a fiscal crisis. Details on the data cross-section are presented in Appendix A. The mapping into the model-implied cross-section is discussed in Appendix B.

Even though parameters are calibrated jointly, some of them load strongly on specific targeted moments: the discount factor $\beta = 0.95$ and average pledgeable share $\overline{\theta} = 0.5$ strongly affect the average debt level and its dispersion in normal times, while $\overline{\theta}^{crisis} = 0.3$ loads on the increase of default risk during a fiscal crisis. Volatility σ_θ and persistence ρ_θ of pledgeable government revenues primarily drive the level and dispersion of spreads. Subsistence consumption $\underline{g}$ generates high levels of default risk at the right tail of the distribution (Bocola et al., 2019), consistent with the large increase in spreads on periphery countries during the European debt crisis.

Lastly, I parametrize investors' utility function over available collateral to match the yield difference of government bonds and the corresponding CDS, which is also referred to as the *CDS-bond basis*, in normal times and during a fiscal crisis. Specifically, the parameter l_0 governs the relative importance of collateral premia and therefore primarily loads on the government bond spread level, once the default risk is matched. The CARA-parameter l_1 determines the elasticity of collateral valuation to aggregate collateral supply and primarily affects the change of bond spreads during crisis periods, in particular for low-risk countries, for which collateral valuation effects are most important.

Table 1: Calibration

Parameter	Value	Source
Persistence income ρ_y	0.76	Euro area data
Vol of income shock σ_y	0.0032	Euro area data
Risk-free rate r^{rf}	0.005	EURIBOR-HCPI
Maturity Parameter π	0.2	5y average maturity
Coupon Parameter c	0.045	Average coupon rate
Haircut Parameter η	0.5	Normalization
Maximum Haircut $\overline{\kappa}$	0.4	Nyborg (2017)
Government discount factor β	0.95	Calibrated
Minimum public goods provision $\underline{g}$	0.2	Calibrated
Average transferable share $\overline{\Theta}$	0.5	Calibrated
Average transferable share $\overline{\Theta}^{crisis}$	0.3	Calibrated
Persistence parameter ρ_θ	0.95	Calibrated
Volatility parameter σ_θ	0.005	Calibrated
Collateral valuation slope l_0	0.003	Calibrated
Collateral valuation curvature l_1	3	Calibrated

The model fit is shown in Table 2. By construction, the fiscal crisis is characterized by higher default risk across the government distribution: CDS-spreads and haircuts for all borrowers increase and this increase is most pronounced for high-risk countries. The higher level of haircuts reduces aggregate collateral supply $\overline{B}$. Therefore, the collateral service of bonds least affected by default risk become more valuable. Formally, since $\frac{\partial l_0 \cdot \exp - l_1 \overline{B}}{\partial \overline{B}} < 0$, investors' *collateral valuation increases*. For the safest bonds available (corresponding to Germany, Finland, and the Netherlands in the data), this effect dominates the effect of elevated default risk: their bond spreads decline, even though their CDS-spreads increase.

For riskier countries, collateral valuation has a smaller effect, because their bonds are subject to larger haircuts. Taken together, this leads to a more dispersed distribution of government bond spreads in a fiscal crisis, consistent with the data.

Table 2: Model Fit

	Normal Times		Fiscal Crisis	
	Data	Model	Data	Model
Debt-GDP(%), $Q_{0.25}$	87	62	89	53
Debt-GDP(%), $Q_{0.75}$	125	142	135	118
Bond-Spread, $Q_{0.25}$	-35	-27	-73	-43
Bond-Spread, $Q_{0.75}$	69	73	340	208
CDS-Spread, $Q_{0.25}$	25	26	45	40
CDS-Spread, $Q_{0.75}$	118	123	359	290

Notes: Bond and CDS-preads are annualized in basis points. For the construction of the cross-section in the data, see Table 5.

3.2 Reconciling Cross-Country Evidence

To corroborate the model's ability to replicate the interaction between sovereign risk, collateral premia, and debt issuance, I use the model-implied cross-section of countries to replicate the cross-country evidence regarding non-pecuniary benefits of government bond holdings and sovereign risk reported by Jiang et al. (2021). Even though their paper interprets the CDS-bond basis more widely as convenience yield, comparing their empirical results to the model-implied cross-section is informative, since in my model (a) collateral premia are the only driver of the CDS-bond basis (their convenience yield measure) and (b) collateral premia are directly linked to fiscal fundamentals via the haircut function. To ease the exposition, define the collateral premium as $l_t^j \equiv (1-\kappa_t^j) \cdot l_0 \cdot e^{-l_1 \overline{B}}$. Using this definition, the following two regressions link country-level *fiscal conditions* to collateral premia:

$$l_t^j - l_t^{DE} = \alpha_0 + \alpha_1 \frac{-(\pi + c)b_t^j + q_t(b_{t+1}^j - (1-\pi)b_t^j)}{y_t^j} + \epsilon_t^j \tag{13}$$

$$l_t^j - l_t^{DE} = \beta_0 + \beta_1 \frac{b_t^j}{y_t^j} + \nu_t^j \tag{14}$$

I draw a sample of 10.000 countries from the stationary equilibrium distribution of the model to estimate eqs. (13) and (14) separately. Equation (13) uses each government's primary surplus, defined as net borrowing minus debt service payments, as explanatory variable for the collateral premium relative to the collateral premium of the safest bond, corresponding to the German bund in the data. Equation (14) uses the debt-to-GDP ratio as proxy of fiscal conditions. In both cases, the *largest* collateral premium will be interpreted as the premium on German government bonds, i.e. $l_t^{DE} \equiv \max_j \{l_t^j\}$. This is replicating the empirical approach of Jiang et al. (2021), who regress time-series averages of bond convenience yields relative to Germany on the average primary deficit and debt-to-GDP ratio across euro area members.

The (untargeted) model-implied effect of fiscal conditions on collateral premia is very similar to the effect reported in Jiang et al. (2021): a one-standard deviation increase in the primary surplus increases the CDS-bond basis by $\hat{\alpha}_1 = 6.6$bp in the data, compared to $\hat{\alpha}_1 = 2.1$bp in the model. Similarly, a one-standard deviation increase in the debt-to-GDP ratio reduces the CDS-bond basis by $\hat{\beta}_1 = 14$bp in the data and by $\hat{\beta}_1 = 22$bp in the model.

3.3 Collateral Policy during a Flight-to-Quality

Using the calibrated model, I examine the extent to which the central bank can affect the spread distribution by adjusting its collateral framework in response to a fiscal crisis. Since the collateral framework in this model is represented by the haircut cap $\overline{\kappa}$, there are two well-defined extreme values. The case of $\overline{\kappa} = 1$ can be interpreted as *strict market discipline* in the sense that investors do not obtain a funding advantage from pledging the government bond with the central bank. When $\overline{\kappa} = 0$, investors can pledge the full market value of every government bond with the central bank.

Table 3 shows that this full collateral backstop is able to partially reduce the yield spread between different countries and thereby mitigates - to some extent - diverging forces in the currency union. This has also an effect on sovereign risk, which slightly decreases in particular for high-risk borrowers. In contrast, the strict market discipline policy slightly reduces collateral supply and at the same time exacerbates the dispersion of bond spreads. Compared to the baseline calibration, there are only negligible on CDS-spreads in this case.

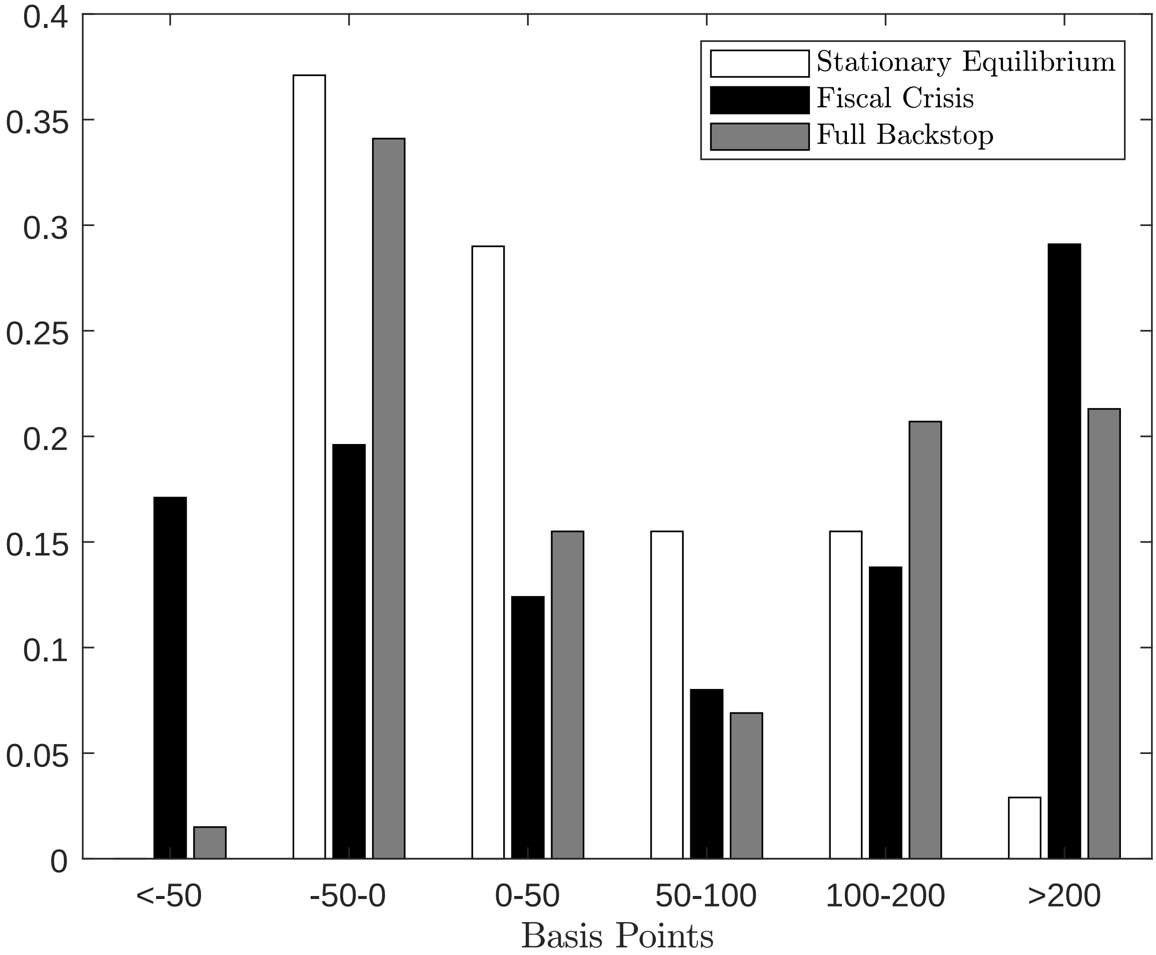
Table 3: Full Collateral Backstop in a Fiscal Crisis

	Baseline	Full Backstop	No Backstop
Collateral Supply (% of GDP)	55.1	65.5	54.8
Bond-Spread, $Q_{0.25}$	-43	-40	-44
Bond-Spread, $Q_{0.75}$	208	194	220
CDS-Spread, $Q_{0.25}$	39	38	39
CDS-Spread, $Q_{0.75}$	290	284	290
Haircut (%), $Q_{0.25}$	11	0	12
Haircut (%), $Q_{0.75}$	40	0	65

Notes: Bond and CDS-preads are annualized in basis points.

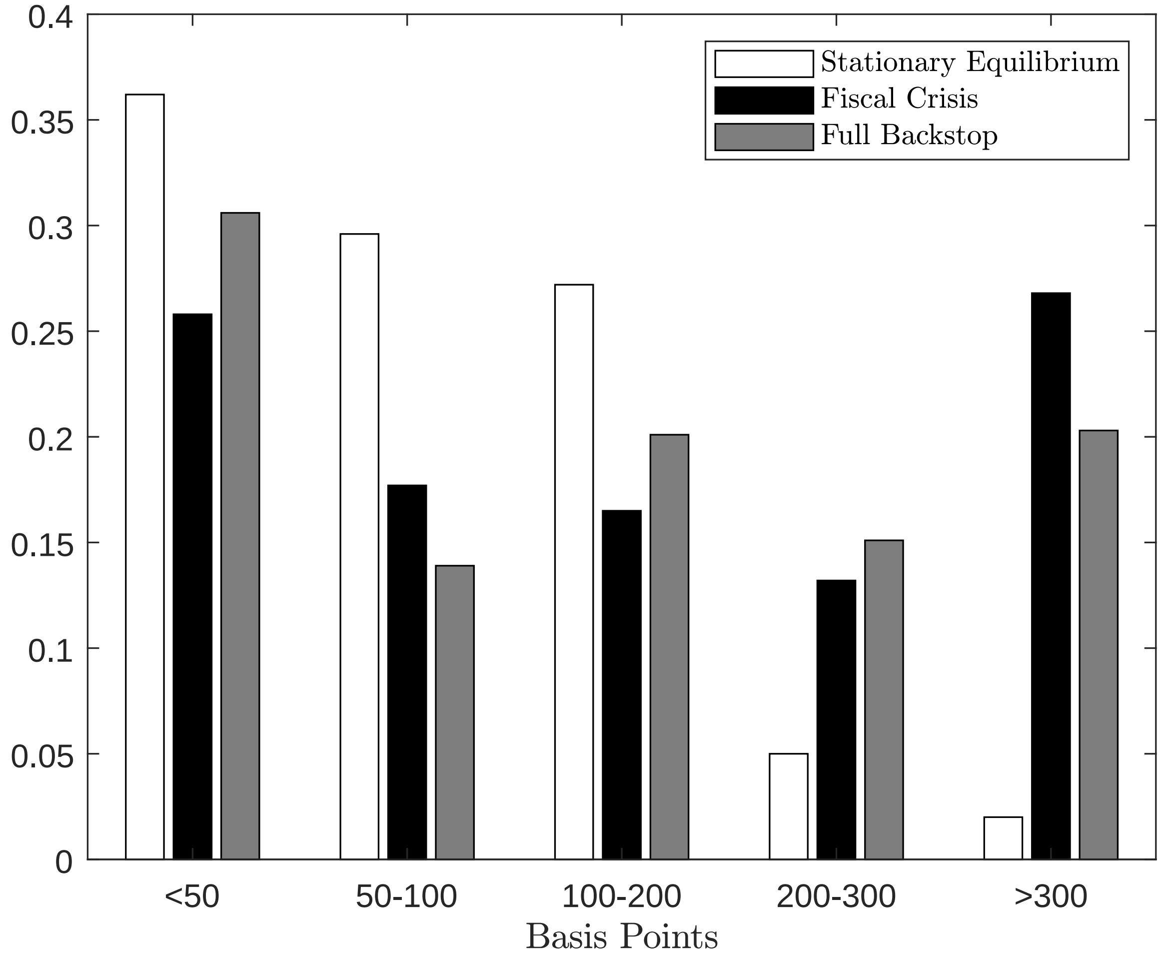
Figure 2 and Figure 3 provide an illustration of the cross-sectional distribution over bond- and CDS-spreads. The stationary equilibrium associated with the baseline calibration is indicated by white bars and shows that most government bonds trade at zero or slightly negative spreads over the risk-free rate. This corresponds to CDS-spreads below 100bp. A small share of borrowers is subject to sizable default risk, with bond- and CDS-spreads above 200bp. The distribution in the fiscal crisis period is indicated by black bars and exhibits a large mass in both the right bond spread bucket (200bp or higher) *and* the low bond spread bucket (-50bp or less). In this case, the CDS-distribution is shifted markedly to the right, with almost 25% of its mass being in the highest CDS-spread bucket.

Figure 2: Equilibrium Bond-Spreads



A full collateral backstop affects the cross-sectional dispersion of spreads in two ways: first, it increases collateral premia on relatively risky countries, since their bonds provide larger collateral service to investors, which has a negative effect on their spreads. The mass of government bonds trading at spreads of 200bp or more drastically reduces, as indicated by gray bars in Figure 2. Second, it decreases the collateral premium on relatively safe countries, since a larger collateral supply reduces investors' *collateral valuation*: the distribution over bond spreads exhibits a much smaller mass in the lowest bucket (-50bp or less). This also reduces the mass of the CDS-distribution in the high-risk bucket, because risky governments are able to roll over their legacy debt stock in the crisis period more easily.

Figure 3: Equilibrium CDS-Spreads



It should again be stressed that the full collateral backstop result is obtained when subjecting the model economy to an *unanticipated fiscal* crisis lasting for a known time horizon. In this setting, policy objectives are shaped by short-run considerations. In the European case, the objectives were twofold: (1) maintaining a sufficiently large collateral supply ensures smooth functioning of financial markets and facilitates monetary policy implementation through the banking sector. (2a) Reducing the government bond spread dispersion between core and peripheral borrowers contains regional imbalances and (2b) avoiding a sovereign default and its adverse consequences. Regional imbalances and sovereign defaults in turn directly threaten currency union' viability. Objective (1) in principle is a standard *lender-of-last-resort* policy, where its implementation is complicated by the elevated default risk of available collateral. Objectives (2a) and (2b) are related to the ECB's role as the *investor-of-last-resort* and are closely related in the sense that reducing the yield dispersion goes hand in hand with reducing the borrowing costs of high-risk countries, which in turn makes a debt roll-over easier for these countries. The model illustrates how a temporary relaxation of collateral policy contributes to achieving these short-term policy objectives using a single instrument.

Notably, this result obtains for an unanticipated aggregate fiscal shock. Studying the cyclical and long-run components of an optimal collateral framework in a monetary union requires a more detailed model of the adverse consequences of operating a permanently lenient collateral framework or of predictably relaxing collateral standards during a fiscal crisis. First, permanently low haircuts can increase the average default risk of individual countries since they make debt issuance permanently cheaper, as shown in Kaldorf and Roettger (2021). Criticism of treating all European government bonds as default-risk free in the lowest haircut tier at the inception of the 'single collateral list' in 2004 dates back to Buiter and Sibert (2005). Similarly, cyclical haircut relaxations can also introduce a moral hazard consideration if governments anticipate that central banks implicitly subsidize their debt rollover during a crisis, which might erode fiscal prudence. In both cases it is necessary to specify why central banks require investors to pledge collateral in the first place and how the costs of a sovereign default are distributed among member states. Likewise, designing optimal LOLR policy with defaultable government debt during a pure banking crisis requires a richer model of the collateral demand side and is potentially intertwined with the government's default behavior if a sovereign-bank nexus is at play. These extensions are left for future research.

4 Conclusion

This paper presented a model with heterogeneous governments issuing bonds subject to default risk. Adding collateral premia to this setting, the model generates a collateralinduced flight-to-quality during a fiscal crisis. Calibrated to the euro area debt crisis, the model can reconcile the cross-sectional distribution of haircuts, government bond spreads, and CDS-spreads. A full collateral backstop policy during crisis periods partially reduces the cross-sectional dispersion of government bond spreads. Moreover, this reduces debt rollover costs for the riskiest countries, which reduces their default risk. These results lend support to fully lenient collateral policy *on a temporary basis* in response to a fiscal crisis, as exemplified by the ECB's decision to temporarily suspend the minimum rating requirement on government bonds in 2020 as a response to the Covid-19 shock.

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A Data Appendix

This section provide an overview of the cross-section of euro area members, before and during the European sovereign debt crisis. The sample is excluding Austria, Cyprus and Luxembourg due to their small size and limited data availability. Government bond spreads are computed from the yield-to-maturity of 5-year benchmark bonds, obtained from *Thomson Reuters Datastream*. The risk-free rate is proxied by 5y-EURIBOR-swaps.

The cross-sectional distribution in the pre-crisis period is shown in Table 4, where the reported spreads are averages from daily data between 2009-01-01 until 2011-06-30. For government bond spreads For debt outstanding and ratings, I use data from 2010Q4 as a cut-off. For all countries except Greece, CDS-spreads are substantially larger than government bond spreads. This so-called 'CDS-bond basis' is largest for the German bund (69bp), still large for Italian (49bp) and Spanish (54bp) bonds and becomes small for the riskiest countries (9bp for Portuguese and 7bp for Irish bonds). The empirical cumulative distribution function of bond- and CDS-spreads is given in the last column, where the weighting corresponds to the market value of debt outstanding. The upper and lower quartile correspond to the German (-35bp) and Italian spread (69bp), respectively.

Table 4: Cross-Section of Euro Area Members, Pre-crisis

Country	Bond-Spread	CDS-Spread	Rating	Debt/GDP (%)	Debt (billions)	Cumulative (%)
Germany	-35	34	AAA	87	2271	26
Finland	-15	35	AAA	55	105	28
Netherlands	-14	33	AAA	70	449	33
France	-12	49	AA	101	2032	56
Belgium	30	89	AA	109	398	61
Italy	69	118	A	124	2018	84
Spain	86	140	A	67	723	93
Portugal	220	229	A	106	139	95
Ireland	291	298	A	83	190	97
Greece	627	570	HY	131	283	100
Total					8609	

Notes: Bond and CDS-preads are annualized in basis points. Countries are sorted by bond spreads in the pre-crisis period. The rating refers to the highest rating by the four recognized credit assessment agencies Moody's, S&P, Fitch, and DRBS, which is in accordance with the ECB collateral framework. Market value of debt outstanding in EUR.

Table 5 shows the data for the crisis period. I use average spreads from 2011-07-01 until 2012-06-30, which is typically considered to be the most severe phase of the European sovereign debt crisis. Debt outstanding refers to 2011Q4. All rating downgrades relevant for collateral valuation occurred in the crisis period. Compared to the pre-crisis period, CDS-spreads increased for all countries, where the increase is most pronounced in the 'periphery' countries (Italy, Spain, Portugal, Ireland and Greece). The rating downgrades in the case of Italy, Spain and Portugal imply larger haircuts on their bonds according to the ECB's collateral framework (Nyborg, 2017). For example, government bonds with a maturity of 5 to 7 years were subject to an 8% haircut after the downgrade, compared to a 3% haircut before.

In contrast, the spreads on German, Finish, and Dutch government bonds show a decline compared to the pre-crisis subsample, which is most pronounced for the German bund.

The CDS-bond basis increases to 118bp for the bund, while it reduces to 19bp (11bp) in the case of Italy (Spain). The CDS-bond basis for Portugal, Ireland, and Greece is less reliable in the crisis period, due to exceptionally large volatilities and market illiquidity.

Table 5: Cross-Section of Euro Area Members, Crisis

Country	Bond-Spread	CDS-Spread	Rating	Debt/GDP (%)	Debt (billions)	Cumulative (%)
Germany	-73	45	AAA	89	2375	25
Finland	-29	60	AAA	63	121	26
Netherlands	-27	66	AAA	79	507	31
France	15	110	AA	112	2290	55
Belgium	109	180	AA	121	450	60
Italy	340	359	BBB	135	2109	82
Spain	326	337	BBB	93	955	92
Portugal	1153	984	BBB	137	228	94
Ireland	558	600	A	129	220	97
Greece	3493	10120	HY	168	310	100
Total				9565		

Notes: Bond and CDS-spreads are annualized in basis points. Countries are sorted by bond spreads in the pre-crisis period. The rating refers to the highest rating by the four recognized credit assessment agencies Moody's, S&P, Fitch, and DRBS, which is in accordance with the ECB collateral framework. Market value of debt outstanding in EUR.

B Numerical Appendix

The model is solved using value function iteration on a discrete grid for government bond holdings with $n_b = 301$ points equally distributed over the grid $[0, 3]$. The idiosyncratic income shock is discretized using the method of Tauchen (1986) on an equispaced grid with $n_y = 101$ points over the interval $[-3\frac{\sigma_\mu}{1-\rho_\mu^2}, 3\frac{\sigma_\mu}{1-\rho_\mu^2}]$. As in Gordon (2018), I use taste shocks over potential debt choices b^j_{t+1} to address the typical convergence issues in this class of model.

To compute the cross-sectional distribution, note that the debt policy function $\mathcal{B}(b^j_t, \theta^j_t, y^j_t)$

defines an endogenous mass-shifter Π_b mapping idiosyncratic states into debt choices. Similar to the computational algorithm in Kaldorf and Wicknig (2021), I use the mode of the distribution over debt choices $\mathcal{B}(b_t^j, \theta_t^j, y_t^j)$ when setting up the mass-shifter. Together with the transition matrices of the transferable revenue share Π_θ and government revenues Π_y, the combined mass shifter is given by $\Pi_g = \Pi_b \otimes \Pi_\theta \otimes \Pi_y$. Π_z is a sparse matrix that implicitly defines the firm distribution G via $G' = G'\Pi_g$. Extracting the distribution thus boils down to computing the right Eigenvalue to Π_g'. This is numerically feasible since Π_g is sparse. Once G is known, collateral supply $\overline{B}$ is obtained by aggregating over the collateral supply $(1-\kappa_t^j)b_{t+1}^j k_t^j$ by individual governments.

Computational Algorithm

1. At iteration step $\iota = 0$, guess government debt policy as $\mathcal{B}(b_t^j, \theta_t^j, y_t^j) = b_t^j$ and compute implied aggregate collateral supply and the bond price schedule.

2. Given the bond price schedule and value function from the previous iteration

   (i) solve the government problem and compute the corresponding endogenous mass shifter Π_b,

   (ii) obtain the mass shifter Π_g and update the cross-sectional distribution G,

   (iii) compute $\overline{B}$ and the associated collateral valuation $l_0 \cdot \exp\left\{-l_1\overline{B}\right\}$,

   (iv) update haircuts and bond prices using the updated debt policy,

   (v) if $\mathcal{V}(b_t^j, \theta_t^j, y_t^j)$ and $q(b_{t+1}^j, \theta_t^j, y_t^j)$ converge, STOP, else go back to (i).

Mapping the Cross-Sectional Distribution to the Data Due to the small number of euro area members making up the cross-section of borrowers to which the model is calibrated, some remarks on the model implied cross-sectional distribution are in order. While in principle, this is an infinite-dimensional object, it becomes finite-dimensional in the process of discretization. To illustrate the mapping from the model into the sample of euro area governments presented in appendix A, consider the left tail of the bond spread distribution, represented in the calibration by the 25%-quantile. As shown in the last column of Table 4, this corresponds to Germany in the data. Since Germany accounted for 26% of the euro area government bond market, I numerically integrate over the interval $[0, 0.26]$ of the model-implied distribution over bond spreads, CDS-spreads, haircuts, and debt outstanding. Likewise, the 75%-quantile is given by Italy in the data and I integrate the model-implied cross-section over the interval $[0.61, 0.84]$.