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Convenient but risky government bonds

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Non-technical summary

Research question

Government bonds are frequently used as collateral to secure financial transactions. The premium that financial market participants are willing to pay for the collateral services of government bonds is typically referred to as *convenience yield*. Thus far, the literature on convenience yield has mostly focused on the United States and highlighted the negative dependence of convenience yield on the supply of government bonds. While default risk is effectively not a concern for the United States, euro area government bonds are subject to sizable default risk. However, they provide non-negligible convenience yield as well. In this paper, we study how convenience yield interacts with the supply of government bonds in the presence of sovereign risk.

Contribution

We build a theoretical model in which convenience yield, the supply of government bonds, and sovereign risk are endogenously and jointly determined. In the model, a government issues defaultable bonds to investors who value them for their collateral services, giving rise to a convenience yield. In the case of risky government bonds, this premium declines not only directly with the supply of government bonds (scarcity channel), but also indirectly via default risk through a novel haircut channel. This channel emerges because larger haircuts are applied to riskier assets if they are pledged as collateral – consistent with practice on the financial markets. We first illustrate the relative importance of these channels for an analytically tractable two-period model and then perform a quantitative evaluation for a calibrated (infinite-horizon) model version.

Results

We show that government bond issuance and default risk are greater than in a model without convenience yield. Furthermore, we demonstrate that the elasticity of convenience yield with respect to debt issuance and default risk is a key component of the analysis. Specifically, a high elasticity leads to fiscal discipline, since each additional unit of debt strongly depresses bond prices. This high elasticity can result from either a highly elastic collateral valuation by investors or from a highly elastic haircut schedule, but not from both at the same time. The reason behind this complementarity is that a larger haircut reduces the collateral value of outstanding government bonds, such that collateral valuation increases. Calibrated to Italian data, we show that our model can replicate empirically observed patterns of sovereign risk, convenience yield, and debt management. Furthermore, convenience yield can account for a sizable share of empirically observed government debt-to-GDP ratios. We also find that highly elastic collateral valuation is consistent with the data. In this case, more elastic haircut schedules reduce fiscal discipline and increase sovereign risk.

Nichttechnische Zusammenfassung

Fragestellung

Zur Besicherung von Transaktionen kommen auf den Finanzmärkten häufig Staatsanleihen zum Einsatz. Für diese Charakteristik sind Investoren bereit eine entsprechende Prämie zu zahlen. Diese Prämie wird typischerweise als "Convenience Yield" bezeichnet. Die Literatur hat sich mit dieser Prämie bisher vor allem im Kontext der USA befasst, für die sich ein negativer Zusammenhang zwischen Convenience Yield und dem Angebot an Staatsanleihen zeigen lässt. Während das Ausfallrisiko bei US-Staatsanleihen vernachlässigbar klein ist, ist dies im Euroraum nicht allgemein der Fall. Trotz teilweise beachtlicher Ausfallrisiken lässt sich jedoch auch dort die Existenz einer Convenience Yield zeigen. Diese Arbeit untersucht die Wechselbeziehung zwischen der Convenience Yield und dem Angebot an Staatsanleihen bei positivem Ausfallrisiko.

Beitrag

Wir entwickeln dazu ein theoretisches Modell, das die Convenience Yield, das Angebot an Staatsanleihen und ihr Ausfallrisiko simultan bestimmt. Im Modell emittiert ein Staat Anleihen, die von Investoren erworben werden und als Sicherheiten verwendet werden können, was zu einer entsprechenden Convenience Yield führt. Staatsanleihen sind jedoch riskant, da der Staat die Rückzahlung seiner Schulden verweigern kann. Höhere Schuldenemission reduziert die Convenience Yield zum einen direkt, da das Anleihenangebot steigt, und zum anderen indirekt aufgrund höherer Bewertungsabschläge („Haircuts"): Auf Finanzmärkten werden höhere Haircuts angewandt, wenn ein zur Besicherung eingesetzter Vermögenswert ein höheres Ausfallrisiko besitzt. Wir arbeiten die relative Bedeutung der beiden Kanäle zunächst mit Hilfe eines analytisch handhabbaren Modells heraus und führen im Anschluss eine quantitative Auswertung für eine kalibrierte Modellversion durch.

Ergebnisse

Unsere Analyse zeigt, dass Staatsschulden und Ausfallrisiko höher sind, wenn Investoren eine Convenience Yield zahlen. Dabei spielt die Elastizität von Convenience Yield bezüglich der Menge der emittierten Schulden und deren Ausfallrisikos eine zentrale Rolle. Die Convenience Yield entfaltet eine disziplinierende Wirkung auf den Staat, wenn zusätzliche Schuldenemissionen den Preis der Anleihen stark reduzieren. Dies funktioniert beispielsweise, wenn die Nachfrage nach Staatsanleihen als Sicherheit sehr sensitiv ist oder Haircuts sehr stark im Ausfallrisiko steigen. Ist jedoch beides gleichzeitig der Fall, entfällt die disziplinierende Wirkung: Höhere Haircuts senken den Wert von Anleihen als Sicherheit, was das Angebot an Sicherheiten drastisch verknappt. Dadurch kann die Nachfrage nach Staatsanleihen sogar ansteigen und kann weitere Anreize zur Schuldenemission setzen. Für unsere quantitative Analyse kalibrieren wir unser Modell auf Basis italienischer Daten und zeigen, dass es die beobachteten Dynamiken von Ausfallrisiko, Convenience Yield und Schuldenmanagement sowie einen beträchtlichen Teil der Schuldenposition erklären kann. Unsere Kalibrierung weist auf eine sehr sensitive Nachfrage nach Staatsanleihen als Sicherheit hin. In diesem Fall reduzieren elastischere Haircuts die fiskalische Disziplin und erhöhen das Ausfallrisiko dieser Anleihen.

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Convenient but risky government bonds*

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Abstract

How does convenience yield interact with sovereign risk and the supply of government bonds? We propose a model of sovereign debt and default in which convenience yield arises because investors are able to pledge government bonds as collateral on financial markets. Convenience yield is dependent on the valuation of collateral, which is negatively dependent on the supply of government bonds, and haircuts that increase with sovereign risk. Calibrated to Italian data, convenience yield contributes substantially to the public debt-to-GDP ratio and can rationalise prolonged periods of negative bond spreads, even in the presence of default risk. We show that the debt elasticity of convenience yield is the most important driver of our results. Decomposing it into the debt elasticity of a collateral valuation and a haircut component, we find that, under empirically relevant conditions, a higher debt elasticity of haircuts can reduce fiscal discipline.

Keywords: Sovereign risk, convenience yield, haircuts, debt management

JEL classification: G12, G15, H63

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1 Introduction

Government bonds play a special role in the financial systems of developed economies. One, if not the most important, reason for this is the exceptional degree of liquidity and safety that they provide to investors in comparison with alternative asset classes. These special attributes make government bonds a key ingredient for the functioning of various financial market segments, such as repo and securities lending markets in which they are posted as collateral. Investors' willingness to pay a price markup for government bonds due to the non-pecuniary benefits that they provide is well documented (for an early reference, see Bansal and Coleman, 1996). The associated premium is usually referred to as "convenience yield".

The literature has documented the negative dependence of convenience yield on government bond supply (Krishnamurthy and Vissing-Jorgensen, 2012), its potential to reconcile the high valuations of government debt (Jiang et al., 2022b; Mian et al., 2022), and its implications for government debt management (Greenwood et al., 2015; Jiang et al., 2022a; Gorton and Ordoñez, 2022). However, the focus so far has usually been on the United States (or Japan), where sovereign risk is effectively not a concern. By contrast, several euro area countries face non-negligible default risk but are still able to issue bonds with sizable convenience yields (Jiang et al., 2021).[1] When sovereign risk becomes a non-negligible component of government bond spreads, studying the implications of convenience yield for the conduct of fiscal policy consequently has to take account of potential interactions between convenience yield, sovereign risk, and the supply of government bonds.[2]

This paper contributes to understanding this interaction, which is a non-trivial task since all of these factors are jointly and endogenously determined in equilibrium. With all else being equal, the presence of convenience yield improves borrowing conditions for a government by raising bond prices, which makes it more attractive for the sovereign to issue debt. However, an increase in the supply of government bonds lowers convenience yield by making bonds less scarce and thus decreases the collateral valuation and convenience yield, ceteris paribus. This *collateral valuation effect* has been documented by Krishnamurthy and Vissing-Jorgensen (2012) and Greenwood et al. (2015) for the United States, i.e. for the case of default-free bonds. In addition, sovereign risk exerts an ambiguous effect on convenience yield, which has not been studied in the literature so far. To cushion against price risk in general and credit risk in particular, the collateral pledged in market transactions is subject to haircuts. The greater the haircut, the less collateral can effectively be pledged. Convenience yield thus negatively depends on the haircut, ceteris paribus. Since haircuts are positively related to a security's credit risk, an increase in sovereign risk adversely affects bond prices not just by raising the default risk premia charged by investors, but also by lowering the convenience yield of government bonds. We refer to this as the *haircut effect*.

The supply of public debt and the risk of default are, however, not determined exogenously, but a reflection of government behaviour, which in turn responds to changes in borrowing conditions and, therefore, convenience yield. How do convenience yield and its two components, *collateral valuation* and *haircuts*, affect a government's incentives to borrow and default? To illustrate the basic interac-

[1] Risky government bonds are still safer and more liquid than most alternative asset classes available to investors. We will therefore focus on this type of security in this paper and abstract from privately issued debt.

[2] Throughout the paper, we measure government bond spreads relative to maturity-matched interest rate swaps.

tions between convenience yield, sovereign risk, and government bond supply, we first propose a simple two-period framework featuring an impatient government that issues defaultable one-period bonds. We assume that investors derive utility from the collateral services of government bonds. In the spirit of Krishnamurthy and Vissing-Jorgensen (2012), the marginal benefit of holding government bonds is negatively dependent on collateral supply, which is given by total outstanding government debt, valued at market prices and corrected for haircuts. Consistent with mark-to-market practice on repo and securities lending markets, haircuts are linked positively to default risk.[3] Investors' maximisation problem yields a pricing schedule for government bonds that is affected by convenience yield in two ways. First, bond prices decrease in debt issuance even absent default risk, since collateral becomes less scarce and convenience yield declines. Second, default risk depresses bond prices and this increase is amplified by mark-to-market haircuts, holding the collateral valuation component constant.

Taking as given this bond pricing schedule and future default incentives, the government chooses bond issuance to maximise public spending over both periods. Absent convenience yield, optimal debt issuance is determined by the slope of the debt issuance Laffer curve (see Arellano, 2008) and the government's relative impatience. Convenience yield drives a wedge into this trade-off and we show analytically that a debt-inelastic convenience yield affects public policy in the same way as investor (im)patience: it increases both debt issuance and default risk. If convenience yield is debt-elastic, the positive effect of convenience yield on debt issuance is dampened and can potentially even be reversed. This observation follows from the fact that a highly debt-elastic convenience yield implies that bond prices are more sensitive to debt issuance as well: the curvature of the Laffer curve increases. The point at which the Laffer curve's slope is equal to relative impatience is therefore lower.

We then provide a decomposition of the elasticity of convenience yield with respect to debt issuance into the elasticity of its components (collateral valuation and haircuts) and illustrate a complementarity between the two components, which will turn out to be of empirical relevance. If the elasticity of collateral valuation with respect to debt issuance is small, a higher responsiveness of haircuts to debt issuance, and thereby to default risk, reduces the government's debt choice. Conversely, if the elasticity of collateral valuation with respect to debt issuance is large, the higher responsiveness of haircuts to debt issuance, and thereby to default risk, increases the government's debt choice. The potential complementarity between collateral scarcity and the haircut effect follows from the fact that collateral valuation depends on the haircut-corrected market value of government bonds. If debt issuance induces a large increase in haircuts and – at the same time – collateral scarcity effects are sufficiently strong, bond prices can *increase* locally. This phenomenon has empirical relevance. For example, it is consistent with the observation that the negative rating outlook on US government bonds in 2011 was accompanied by a decline in bond yields. In our model, a government exploits such collateral scarcity effects and optimally chooses higher debt issuance in this constellation.

To assess the relevance of convenience yield and its components, we embed them into a quantitative model of sovereign debt and default (see Chatterjee and Eyigungor, 2012; Bocola et al., 2019). In the

[3] In practise, the effective convenience yield of a bond depends on the haircut to which it is subject to in various market segments. In the model, when using the term *haircut*, we refer to an average haircut over different segments. For example, Drechsler et al. (2016) document that the ECB applied a flatter haircut schedule for its standing facilities than the haircuts set by a large central clearing counterparty (CCP). The average haircut encompasses this heterogeneity in a tractable way. For our comparative statics exercises with respect to the steepness of haircut schedules, we do not take a stance on whether it reflects the risk management practice of CCPs or central bank collateral policies (see Nyborg, 2017).

model, a risk-averse government faces persistent revenue shocks and smooths public spending by issuing long-term bonds to investors. As in the simple two-period model, the government lacks commitment and convenience yield enters the model through investors' collateral valuation. The quantitative model is calibrated to Italian data for the time period from 2001 to 2012.

Our main variables of interest are the government bond spread s, which we compute over maturity-matched interest rate swaps as a risk-free rate proxy in the data, and convenience yield. Since the bond spread solely reflects credit risk and convenience yield in our model, we use credit default swap (cds) spreads as a measure of credit risk, since derivatives do not provide convenience yield. Consequently, we directly measure convenience yield as the CDS-bond basis ($cy \equiv s - cds$).[4] Notably, we do not need to assume that CDS spreads are a pure measure of default risk in the data as long as the CDS-bond basis is correlated with collateral supply and thus convenience yield.[5]

Free model parameters are calibrated to match the mean and volatility of bond spreads, CDS spreads, and debt outstanding over the full sample. Although the model is quite parsimonious, it picks up several important (non-targeted) features of the crisis episodes observed in the data. While government bond spreads are negative during safe times in the data, they turn positive in 2008Q4, which prompts us to interpret the time period from 2008Q4 to 2012Q4 as the crisis sub-sample. When applying this filter to model-generated time series, we find that the model-based crisis sub-sample exhibits higher spread volatility and a moderate increase in debt issuance, which is consistent with the data. We can therefore broadly distinguish between two endogenous regimes. In times of crisis, default risk is found to dominate the convenience yield component of the bond spread, whereas the opposite is true for safe times, during which bond spreads are usually negative. This is conceptually distinct from a model with risk-averse investors, which can rationalise high government bond spreads during a fiscal crisis using risk premia (for a discussion, see Lizarazo, 2013), but would not be capable of generating negative bond spreads in safe times.

We also show that, in our baseline calibration, the model is able to match key fiscal policy moments, such as the countercyclicality of debt issuance. It also generates a highly left-skewed distribution of bond and CDS spreads. To further corroborate the validity of our calibrated model, we investigate the role played by government bond supply for convenience yield. Specifically, we use the simulated time series to regress the convenience yield on bond supply. Both the data and the model-implied regressions reveal a highly significant negative effect of bond supply. This suggests that collateral valuation is empirically relevant in the presence of default risk, which complements the findings of Jiang et al. (2021), who identify convenience yield in a panel of risky European government bonds.

Using our calibrated model, we demonstrate that convenience yield can account for an economically relevant part of the public debt-to-GDP ratio. Without convenience yield, the debt-to-GDP ratio is 24% lower than in the baseline after recalibrating the model to match the average CDS spread. Alternatively, the government would have to be substantially more impatient to match the observed debt-to-GDP ratio.

[4]This is slightly different to Jiang et al. (2021), who analyse the difference between the CDS-bond basis of risky European borrowers and the German CDS-bond basis.

[5]A potentially important risk factor in CDS spreads is counterparty risk, i.e. the uncertainty about CDS payoffs in the event of a sovereign default (for a discussion, see Salomao, 2017). In this setting, CDS spreads increase to a degree of less than one-for-one in expected default risk, since they do not provide perfect insurance against default. In this case, CDS spreads underestimate actual default risk, such that the increase in convenience yield during times of fiscal stress is estimated conservatively.

We then demonstrate that the elasticity of convenience yield with respect to debt issuance drives this result. To do so, we make collateral valuation a constant, but keep the haircut schedule in its baseline form. In this case, the debt-to-GDP ratio declines by 21% once the model is recalibrated to match the average CDS spread and the average convenience yield. The shape of convenience yield, rather than its level, is more important for sovereign debt and default risk, which is in line with recent results by Mian et al. (2022) for default risk-free government bonds.

In another experiment, we show that the complementarity between collateral valuation and haircut components also carries through to a more elaborate setting. In the baseline calibration, a reduction in the elasticity of the haircut schedule translates into a reduction in average CDS spreads and their volatility. The long-run default rate declines as well. This is due to the disciplinary impact of collateral scarcity resulting from more debt-elastic bond pricing. However, if collateral valuation is constant, the same reduction in the elasticity of haircut schedules translates into a higher and more volatile CDS spread and a higher frequency of default on average. Taking account of the endogeneity of government bond supply and default risk thus has potentially important implications for the design of haircuts, both for private and public market segments.

In addition to default risk and convenience yield, the literature has identified market illiquidity as a potentially important determinant of bond pricing (He and Milbradt, 2014; Pelizzon et al., 2016). It is important to note that government bond markets are typically characterised by exceptionally high liquidity, such that bid-ask spreads are usually very low during safe times.[6] However, illiquidity discounts increase with credit risk (see Chaumont, 2018; Passadore and Xu, 2022) and can therefore be a distinct channel that might amplify the widening of government bond spreads in times of high default risk. In the quantitative model, we address the concern that interactions between debt, default risk and convenience yield are, in fact, driven by market illiquidity as a confounding factor by explicitly adding it to the model. To do so, we introduce trading frictions as in Lagos and Rocheteau (2009) into our model. Our interpretation of market illiquidity reflects the notion that investors face trading frictions when selling or buying a security on secondary markets, resulting in illiquidity discounts that are usually measured via bid-ask spreads. Our quantitative results are barely affected by endogenising market illiquidity. We also show that our results do not depend critically on time-varying collateral demand, which might, for example, increase during periods of high financial stress.

Related literature. Our paper draws on the quantitative sovereign default literature (see Aguiar et al., 2016), which mostly focuses on external public debt and emerging economies. Similar to Bocola et al. (2019), we reinterpret the established sovereign default framework à la Eaton and Gersovitz (1981) and Arellano (2008), as a relationship between a government and lenders, regardless of their place of residence, and applies this to a developed economy.[7] Perez (2018) analyses the role of government bonds as public liquidity, which provides additional repayment incentives in a small open economy model. In contrast to that paper, we assume that investors do not enter the government objective, which in our view is a more plausible assumption in light of our application to the euro area.

[6] Bid-ask spreads are a common measure of market illiquidity, whose interaction with bond spreads is well established (He and Milbradt, 2014).

[7] Bocola et al. (2019) show that total government debt, rather than external government debt, is of significance for default risk, which is captured by our model.

Our paper is also related to recent studies that emphasise the role of convenience yield for optimal debt management. Canzoneri et al. (2016) study optimal Ramsey monetary and fiscal policy when government bonds provide transaction services, leading to a trade-off between tax smoothing and liquidity provision (see also Angeletos et al., 2023). Jiang et al. (2022a) analyse convenience yield in a model in which the government trades off insuring bondholders and taxpayers. By contrast, Greenwood et al. (2015) study optimal debt maturity management when short-term government bonds are valued by investors for having money-like properties. While these papers consider the interaction between public debt and the non-pecuniary benefits of government bonds from an optimal policy perspective, their focus is on analytical results and – more importantly – they abstract from sovereign risk, market illiquidity, and lack of commitment.[8]

Layout. The paper is structured as follows. In Section 2, we introduce a simple model of convenience yield and sovereign risk. Section 3 presents the infinite horizon model, which we calibrate in Section 4. Section 5 discusses the interactions between convenience yield, credit risk, and government bond supply. Section 6 shows that our quantitative results are robust to various extensions. Section 7 concludes.

2 A simple model of default risk, convenience yield, and haircuts

In this section, we illustrate the basic interactions between sovereign risk, convenience yield and debt issuance in a two-period setting. In the model, convenience yield introduces a wedge into investors' demand condition for government bonds. The elasticity of this wedge with respect to debt issuance shapes the effect of convenience yield on debt issuance and default risk. Under a standard monotone hazard rate assumption on public revenues, we show that, if convenience yield is debt-inelastic, the government issues more bonds than it would in the absence of convenience yield. Introducing a debt-elastic convenience yield can overturn this result. We then decompose convenience yield into a pure collateral valuation component and a haircut component and derive conditions under which the elasticity of both components increases government debt issuance and default rates.

Environment. At time $t = 0$, an impatient and risk-neutral government (with discount factor $\beta < 1$) issues bonds at price q to a mass-one continuum of competitive investors who do not discount the future. At $t = 1$, the government draws a revenue shock τ with cumulative density function $F(\tau)$ and defaults if debt outstanding b exceeds government revenues. The default probability associated with debt choice b is therefore simply given by $F(b)$. In the event of default, there is no debt recovery for investors and the government does not consume anything.

Investors are risk-neutral but also draw non-pecuniary benefits from holding government bonds due to their eligibility as collateral, given by the function $u(\theta)$. We assume that this *collateral valuation* function satisfies $u'(\cdot)$, $-u''(\cdot) \geq 0$ and that $\theta \equiv (1-\kappa(b))(1-F(b))b$, the *haircut-adjusted* expected payoff, is relevant for the utility investors gain from holding government bonds.[9] The implicit assump-

[8] Bonam (2020) studies the implications of convenience yield for fiscal policy in a New Keynesian setting that models fiscal policy via exogenous policy rules and abstracts from sovereign risk.

[9] One potential micro-foundation for the collateral valuation function is uninsurable liquidity shocks that investors can settle cheaply by borrowing against collateral or by using more expensive unsecured borrowing. The amount that can be borrowed at

tion behind the dependence of the valuation function $u(\cdot)$ on collateral θ is that investors value collateral services at the beginning of period $t=1$, before tax revenues are realised and the government defaults. In this case, collateral depends on the market price of debt at that time, which equals $1-F(b)$, as well as the haircut adjustment $1-\kappa(b)$.[10] We furthermore assume that the consumption of investors does not enter the government's objective, either because they reside outside the economy or because domestic agents can trade with foreign investors on international financial markets in a frictionless manner.[11]

The presence of collateral valuation $u(\cdot)$ gives rise to a convenience yield $\Lambda(b)$ in the government bond pricing equation, which is given by

$$q(b) = \big(1-F(b)\big)\big(1+\Lambda(b)\big) \, .$$

Convenience yield $\Lambda(b) = (1-\kappa(b))u'(\theta)$ in turn depends on the pure collateral valuation component $u'(\theta)$ and the haircut component $1-\kappa(b)$. Throughout the analysis, we assume that haircuts are increasing in debt issuance $\frac{d\kappa(b)}{db} > 0$. In practice, as well as in the quantitative model with long-term bonds and persistent revenue shocks analysed later, haircuts are related to default risk. However, in the two-period setting, default risk $F(b)$ is immediately determined by debt issuance b, such that we express the dependence of haircuts on the government's policy directly in terms of its debt choice.

Debt issuance. Taking as given investors' bond pricing condition, the government maximises

$$\max_{b} \quad q(b)b + \beta \int_{b}^{\infty} (\tau - b) dF(\tau) \, .$$

Throughout this section, we will refer to the relationship between debt b and bond revenues raised in that period, $q(b)b$, as the *debt issuance Laffer curve*. The first-order condition requires that the government issues debt up to the point at which expected repayment obligations (per unit of debt b) equal the slope of the debt issuance Laffer curve:

$$\beta\big(1-F(b)\big) = q(b) + \frac{dq(b)}{db} b \, . \tag{1}$$

Without convenience yield, the bond price simply reflects expected repayment $q(b) = 1-F(b)$, and its derivative is given by $\frac{dq(b)}{db} = f(b) \equiv \frac{dF(b)}{db}$. Let the hazard rate of government revenues τ be denoted as $h(\tau) \equiv \frac{f(\tau)}{1-F(\tau)}$. Plugging this definition, together with the bond pricing expression, into the first-order condition (1), optimal debt issuance absent convenience yield, denoted as b^0, is implicitly defined through

$$h(b^0)b^0 = 1-\beta \, .$$

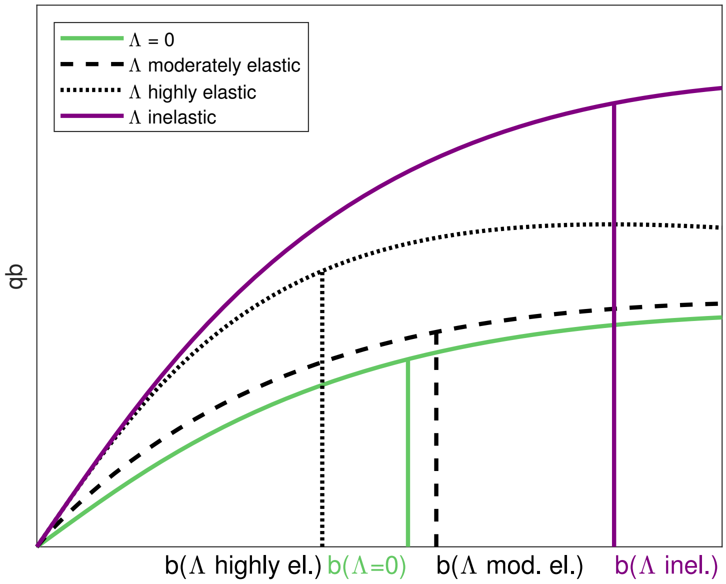
favourable conditions is limited to the market value of government bonds corrected for haircuts. The collateral valuation then declines in haircuts, since they reduce the amount that can be borrowed in the secured segment.

[10] For the quantitative model, we also consider an extension in which we model in detail the interaction between investors at the beginning of the period based on a decentralised market.

[11] Under this assumption, the total debt stock becomes the relevant state variable that determines sovereign risk. Therefore, the supply of government bonds in the model corresponds to total public debt rather than total external debt. For a detailed discussion, we refer to Bocola et al. (2019).

The left-hand side, $h(b^0)b^0$, can be interpreted as the elasticity of default losses with respect to debt issuance. Assuming monotonicity of the hazard rate $h(\cdot)$, the debt choice b^0 is pinned down uniquely.

Figure 1: The effect of convenience yield on debt issuance



The role of convenience yield. The bond price schedule with convenience yield is given by $q(b) = \big(1-F(b)\big)\big(1+(1-\kappa(b))u'(\theta)\big)$, while its derivative reads $\frac{dq(b)}{db} = -(1+\Lambda(b))f(b) + \big(1-F(b)\big)\frac{d\Lambda(b)}{db}$. Both objects, the bond price and its derivative, are larger in absolute terms than in the case without convenience yield. This implies that, in the risky borrowing region, each additional unit of debt *(i)* increases public consumption in the first period to a larger degree than in the absence of convenience yield, and *(ii)* has a more pronounced negative effect on the bond price. With these two competing effects, the first-order condition for debt issuance is given by

$$\left[(1+\Lambda(b))f(b) - \big(1-F(b)\big)\frac{d\Lambda(b)}{db}\right]b = \big(1-F(b)\big)(1-\beta+\Lambda(b))$$

Expressing this in terms of the hazard rate, we have

$$h(b)b = \frac{1-\beta+\Lambda(b)+b\frac{d\Lambda}{db}}{1+\Lambda(b)}\,. \tag{2}$$

While the left-hand side is still given by the elasticity of default losses with respect to debt issuance, the right-hand side now also contains a convenience yield wedge in addition to relative impatience $1-\beta$. Since the sign of marginal convenience yield $\frac{d\Lambda(b)}{db}$ is not known in the general case, convenience yield has an ambiguous effect on debt issuance. In the special case of a debt-inelastic convenience yield $\widetilde{\Lambda}$, the debt choice (2) reduces to

$$h(\widetilde{b})\widetilde{b} = \frac{1-\beta+\widetilde{\Lambda}}{1+\widetilde{\Lambda}}\,. \tag{3}$$

In this special case, convenience yield effectively makes the government more impatient and thereby induces more debt issuance and default risk ($\widetilde{b} > b^0$) through the monotone hazard rate assumption on government revenues. This is reflected by the solid purple line in Figure 1, which depicts the government's debt issuance Laffer curve, i.e. the amount of resources $q(b)b$ raised by issuing b units of debt. With a debt-inelastic convenience yield, $q(b)b$ is higher for all b than in the case without convenience yield (solid green line in Figure 1). In each case, the optimal debt levels are indicated by the vertical lines. Consistent with the first-order condition (2), the elasticity of default losses $h(b)b$ with respect to debt issuance equals relative impatience, potentially adjusted for convenience yield.

The intermediate cases with a debt elastic convenience yield are reflected by the dashed and dotted lines in Figure 1. We consider two cases. The dashed black line corresponds to the case of a relatively small, but moderately debt-elastic convenience yield. The location and shape of the Laffer curve resemble the case without convenience yield and the debt choice increases slightly. By contrast, the dotted black line reflects a large and highly elastic convenience yield. For low levels of default risk, the Laffer curve is almost parallel to the case of inelastic convenience yield. Once default risk becomes greater, it flattens sharply, such that the optimal debt choice is even lower than in the case without any convenience yield.

The role of haircuts and collateral scarcity. Since the responsiveness of the convenience yield to debt issuance is pivotal in determining the government's debt choice, the remainder of this section provides a decomposition of $\frac{d\Lambda(b)}{db}$ into a collateral scarcity effect and a haircut effect. Specifically, marginal convenience yield $\frac{d\Lambda(b)}{db}$ can be expressed as

$$\frac{d\Lambda(b)}{db} = \underbrace{(1-\kappa(b))u''(\theta)\frac{d\theta(b)}{db}}_{\text{Collateral scarcity effect}} + \underbrace{\frac{d(1-\kappa(b))}{db}u'(\theta)}_{\text{Haircut effect}} . \tag{4}$$

While the haircut effect is negative by construction, the collateral scarcity effect has an ambiguous sign. Keeping haircuts and bond prices constant, for example in a model with risk-free debt, more debt issuance makes government bonds less scarce and reduces convenience yield. In the model with default risk, debt issuance can reduce the haircut-corrected market value of government bonds if haircuts are very sensitive to debt issuance and increases in collateral scarcity. Formally, this effect is given by

$$\frac{d\theta(b)}{db} = \big(1-F(b)\big)\left[(1-\kappa(b))\big(1-h(b)b\big) + \frac{d(1-\kappa(b))}{db}b\right] . \tag{5}$$

To relate this potential ambiguity to the debt choice, we use the observation that marginal convenience yield $\frac{d\Lambda(b)}{db}$ only enters the first-order condition (2) together with debt issuance b. It is therefore possible to express the optimal debt choice with convenience yield in closed form. Denoting the elasticity of investors' marginal utility from holding government bonds by $R(\theta) \equiv \frac{-\theta u''(\theta)}{u'(\theta)}$ and the elasticity of the collateral value $1-\kappa(b)$ with respect to debt issuance by $\varepsilon_{\overline{\kappa}}(b)$, optimal debt issuance can be written as

$$h(b^*)b^* = \frac{1-\beta+\Lambda(b^*)\big(1+\varepsilon_{\overline{\kappa}}(b^*)-R(\theta^*)\big)-R(\theta^*)\varepsilon_{\overline{\kappa}}(b^*)\big)}{1+\Lambda(b^*)\big(1-R(\theta^*)\big)} . \tag{6}$$

where $\theta^* = (1-\kappa(b^*))(1-F(b^*))b^*$ is haircut-corrected debt outstanding, valued at the expected payoff.[12] The elasticity of convenience yield, which affects the debt choice in addition to its level, can be directly related to the curvature of the collateral valuation function $u(\cdot)$ and the curvature of the haircut function $\kappa(b)$. In isolation, each component of convenience yield provides disciplinary incentives to the government. However, their interaction term gives rise to non-monotonic interactions, since the government strategically takes into account collateral scarcity effects on the pricing of its bonds.

Two special cases. To provide more intuition for this result, it is useful to consider several special cases of (6). For a *debt-inelastic* collateral valuation component, the debt choice (6) simplifies to

$$h(\overline{b})\overline{b} = \frac{1-\beta+\Lambda(\overline{b})\big(1+\varepsilon_{\overline{\kappa}}(\overline{b})\big)}{1+\Lambda(\overline{b})} \,. \tag{7}$$

If the elasticity of the haircut component with respect to debt issuance, $\varepsilon_{\overline{\kappa}}(\overline{b})$, is sufficiently large (in absolute terms), convenience yield will provide disciplinary incentives to the government, counteracting the basic positive effect of convenience yield on debt issuance described in equation (3). By contrast, if the elasticity of $u'(\cdot)$ is set to a constant $\widehat{R}$, we obtain

$$h(\widehat{b})\widehat{b} = \frac{1-\beta+\Lambda(\widehat{b})\big(1-\widehat{R}\big)(1+\varepsilon_{\overline{\kappa}}(\widehat{b}))\big)}{1+\Lambda(\widehat{b})\big(1-\widehat{R}\big)} \,. \tag{8}$$

It is possible to distinguish between two cases. If $\overline{R} < 1$, collateral scarcity and haircuts are *substitutes* in the sense that both components provide incentives for the government to reduce debt issuance. The debt-inelastic case is nested as the special case $\overline{R} = 0$. By contrast, collateral valuation itself will provide incentives to reduce debt issuance if its curvature is sufficiently high ($\widehat{R} > 1$), which holds even with constant haircuts ($\varepsilon_{\overline{\kappa}}(b) = 0$ for all b). In this case, there is a *complementarity* between both components and a greater haircut elasticity increases debt issuance and default risk. The reason behind this is that haircuts also affect collateral valuation indirectly through their effect on θ (see equation (5)). If larger

[12] We can rewrite marginal convenience yield (4) as

$$b\frac{d\Lambda(b)}{db} = b(1-\kappa(b))u''(\theta)\frac{d\theta}{db} + b\frac{d(1-\kappa(b))}{db}u'(\theta) \,,$$

and plug in (5)

$$b\frac{d\Lambda(b)}{db} = b(1-\kappa(b))u''(\theta)\big(1-F(b)\big)\Big[(1-\kappa(b))\big(1-h(b)b\big) + \frac{d(1-\kappa(b))}{db}b\Big] + b\frac{d(1-\kappa(b))}{db}u'(\theta) \,.$$

We can factor out $\big(1-\kappa(b)\big)u'(\theta)$ and use the definitions of $\theta = (1-\kappa(b))(1-F(b))b$ and $\varepsilon_{\overline{\kappa}}(b) = \frac{b\frac{d(1-\kappa(b))}{db}}{(1-\kappa(b))}$ to write

$$b\frac{d\Lambda(b)}{db} = \big(1-\kappa(b)\big)u'(\theta) \times \left\{\frac{u''(\theta)}{u'(\theta)}\theta\Big(\big(1-h(b)b\big)+\varepsilon_{\overline{\kappa}}(b)\Big) + \varepsilon_{\overline{\kappa}}(b)\right\} \,.$$

Together with the definitions of convenience yield $\Lambda(b) = \big(1-\kappa(b)\big)u'(\theta)$ and of the elasticity of marginal utility $R(\theta) = \frac{-\theta u''(\theta)}{u'(\theta)}$, we can plug this into the first-order condition (2) to obtain

$$h(b^*)b^* = \frac{1+\Lambda(b^*)+\Lambda(b^*)\times\Big\{-R(\theta^*)\Big(\big(1-h(b^*)b^*\big)+\varepsilon_{\overline{\kappa}}(b^*)\Big)+\varepsilon_{\overline{\kappa}}(b^*)\Big\}-\beta}{1+\Lambda(b^*)} \,.$$

Rearranging for $h(b^*)b^*$ yields (6).

haircuts induce a sufficiently great reduction in available collateral θ and if this reduction at the same time induces a sufficiently large collateral scarcity effect ($u''(\theta)$ large in absolute terms), bond prices can even increase locally in response to debt issuance. This situation is consistent with the effects that the negative rating outlook had on US borrowing conditions in 2011. In this case, the optimal government response increases debt issuance.

In the following, we will use a more elaborate setting with long-term debt, strategic sovereign default, and positive debt recovery to assess whether convenience yield itself and the non-monotonic interaction of collateral valuation and haircut components have a quantitatively relevant impact on sovereign debt and default risk.

3 Quantitative model of convenience yield and endogenous default

Time is discrete and denoted as $t = 0, 1, 2, ..., \infty$. The model features a *government* of an economy that issues bonds to a mass-one continuum of ex ante homogeneous *investors*. The government receives random tax revenues, cares about the supply of a public good, and can trade long-term government bonds with investors. Following the quantitative sovereign default literature, the government cannot commit to future repayment and debt issuance (see Arellano, 2008). Investors value government debt for their pecuniary future benefits as well as their *collateral services*, such that they are willing to pay a premium on government bonds. Following Krishnamurthy and Vissing-Jorgensen (2012) and Gorton and Ordoñez (2022), we refer to this valuation of collateral services as *convenience yield*.

We first show the investor problem, which yields a pricing schedule for government bonds. We then describe the government's policy problem, taking as given the pricing schedule, which, together with bond market clearing, characterises the equilibrium of the model.

Investors. Investors receive a large, constant endowment e, i.e. they have deep pockets and discount consumption at the exogenously given, time-invariant rate $r^{rf} > 0$. They maximise their expected lifetime utility

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \frac{c_t + u(\theta_t)}{(1+r^{rf})^t} \right]$$

where c_t denotes an investor's consumption and $u(\theta_t)$ is the convenience benefit of holding government bonds. The collateral value of government bonds $\theta_t \equiv (1-\kappa_t)m_t b_{t-1}$ consists of three parts. The first part is the stock of government bonds b_{t-1} held at the beginning of period t. The second part is given by the expected bond payoff $m_t \equiv \mathbb{E}_{t-1}[k_t]$, with k_t being the cum-coupon bond payoff in period t.[13] The third part is the haircut κ_t imposed on government bonds, which depends on the credit risk of the bond (see further below). Letting the collateral value θ_t depend on the cum-coupon bond price is consistent with mark-to-market practices on repo and securities lending markets. The first-order condition of the

[13] As in the two-period model, we assume that investors value collateral services at the beginning of each period, before information about tax revenues and government bond payoffs arrives. Therefore, we condition the expectations operator on the information set available to investors at the end of $t-1$.

investor problem yields the bond pricing equation

$$q_t = \frac{1}{1+r^{rf}} \times \underbrace{m_{t+1}}_{\text{Payoff}} \times \underbrace{(1+\Lambda_{t+1})}_{\text{Convenience yield}} \quad , \tag{9}$$

where convenience yield consists of two components, associated with haircuts and collateral valuation, respectively:

$$\Lambda_{t+1} \equiv \underbrace{(1-\kappa_{t+1})}_{1-\text{Haircut}} \times \underbrace{u'(\theta_{t+1})}_{\text{Coll. valuation}} \quad . \tag{10}$$

Government. Each period, the government receives exogenous revenues τy_t, where τ is a constant income tax rate and y_t is random domestic income, supplies the public good g_t, issues long-term bonds and decides whether to repay its creditors in order to maximise

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t v(g_t) \right], \quad \beta \in (0,1), \, v'(\cdot) > 0, \, v''(\cdot) < 0,$$

subject to the period budget constraint

$$g_t = \tau y_t + h_t \times \left(q_t \left(B_t - (1-\delta) B_{t-1} \right) - \widetilde{\delta} B_{t-1} \right) \, ,$$

where B_{t-1} denotes legacy debt at the beginning of period t. The budget constraint and the bond payoff depend on the government's credit status h_t. In the autarky case ($h_t = 0$), the government finances the public good g_t with revenues τy_t only. In the repayment case ($h_t = 1$), it can issue new bonds, but has to make debt payments.

Following Chatterjee and Eyigungor (2012), government bonds are modelled as random-maturity bonds. An outstanding bond matures with constant probability δ, where $0 < \delta \leq 1$, whereas it does not mature with probability $1-\delta$, pays a fixed coupon χ, and is valued – like newly issued bonds – at current market price q_t. The per unit bond payoff k_t depends on the default decision d_t of the issuing government, where $d_t = 1$ if the government defaults and $d_t = 0$ if it repays.

When entering a period with a good credit status $h_{t-1} = 1$, a default immediately changes the government's credit status to $h_t = 0$. By contrast, if the government enters a period in which $h_{t-1} = 0$, with constant probability ϑ, it is given the offer to repay the constant fraction $\omega \in [0,1]$ of its debt and immediately leave autarky in return. The indicator variable $\xi_t \in \{0,1\}$ denotes whether such an offer is received ($\xi_t = 1$) or not ($\xi_t = 0$). Following Hatchondo et al. (2016), if the government does not accept an offer ($d_t = 1$), it remains in autarky but its bond position is still reduced by $1-\omega$. If the government declines an offer, it will be excluded ($h_t = 0$) until the next period, in which, with probability ϑ, it might get a new chance to settle its debt and leave autarky. The law of motion for the credit status can therefore conveniently be written as

$$h_t = \xi_t (1-d_t)(1-h_{t-1}) + (1-d_t) h_{t-1},$$

with the (realised) bond payoff given by

$$k_t = \mathbf{1}\{h_{t-1} = 1 \wedge d_t = 0\} \cdot \left(\widetilde{\delta} + (1-\delta)\, q_t\right) + \mathbf{1}\{(h_{t-1} = 1 \wedge d_t = 1) \vee (h_{t-1} = 1 \wedge \xi_t = 0)\} \cdot q_t$$
$$+\mathbf{1}\{h_{t-1} = 0 \wedge \xi_t = 1 \wedge d_t = 1\} \cdot \omega q_t + \mathbf{1}\{h_{t-1} = 0 \wedge \xi_t = 1 \wedge d_t = 0\} \cdot \omega(\widetilde{\delta} + (1-\delta)\, q_t),$$

where $\widetilde{\delta} \equiv \delta + (1-\delta)\chi$. Bond issuance is therefore given by $B_t - (1-\delta)B_{t-1}$, while $\widetilde{\delta}B_{t-1}$ is the period t debt service. The government is assumed to lack the ability to commit to future debt and default policies. Following Bianchi et al. (2018), a default leads to utility costs $\phi(\tau y_t) \geq 0$, which are a function of income, and exclusion from financial markets until debt repayment is settled.[14]

Policy problem. As it is common in the literature, we focus on Markov-perfect equilibria, such that government policy in a given period t only depends on the payoff-relevant aggregate state variables, which consist of the aggregate public debt position B_{t-1}, tax revenues y_t, the government's credit status h_{t-1}, and the offer indicator ξ_t. We present the policy problem recursively, with the next period's variables indicated by a prime.

In the repayment case, assuming bond market clearing ($b = B$), the government faces the following bond pricing schedule for an arbitrary debt choice B', and current income y:

$$q(B', y) = \frac{1}{1+r^{rf}} m^r(B', y) \left(1 + \Lambda^r(B', y)\right) . \tag{11}$$

The bond price schedule consists of two terms. The first one, given by the function

$$m^r(B', y) = \mathbb{E}_{y'|y}\left[\ (1 - \mathcal{D}(B', y')) \left(\widetilde{\delta} + (1-\delta)\mathcal{Q}^r(B', y')\right) + \mathcal{D}(B', y')\mathcal{Q}^d(B', y') \ \right] ,$$

captures the expected pecuniary value of the bond in the next period, where $\mathcal{D}(\cdot)$ denotes the default policy function. Bond prices exceed the expected pecuniary value, which is reflected in the convenience yield

$$\Lambda^r(B', y) = \left(1 - \kappa\left(\lambda(B', y), 1\right)\right) \times u'\left(\Theta^r\left(B', y\right)\right) .$$

Here, the term

$$\Theta^r\left(B', y\right) = \left(1 - \kappa\left(\lambda(B', y), 1\right)\right) \times m^r(B', y) \times B',$$

is the effective collateral service, which depends on the government's debt issuance B'. The haircut $\kappa(\lambda(B', y), 1)$ only depends on the default probability and the government's credit status. The default probability $\lambda(B', y)$ follows from the one-period-ahead default policy

$$\lambda(B', y) = \mathbb{E}_{y'|y}\left[\mathcal{D}(B', y')\right] .$$

The functions $\mathcal{Q}^r(\cdot)$ and $\mathcal{Q}^d(\cdot)$ determine the equilibrium bond prices in the next period for the repayment and default case. The equilibrium bond price in the repayment case is obtained by evaluating the

[14] Public goods provision thus equals public revenues in periods of default and autarky. Therefore, there is no direct output loss in our model that can be related to GDP.

pricing schedule at the debt policy function $\mathcal{B}(\cdot)$

$$\mathcal{Q}^r(B,y) = q\left(\mathcal{B}(B,y),y\right) \ . \tag{12}$$

In the default (and autarky) case, debt service is suspended and the debt position is rolled over, such that the associated equilibrium bond price is given as

$$\mathcal{Q}^d(B,y) = \frac{1}{1+r^{rf}}\left(m^d(B,y)(1+\Lambda^d(B,y))\right) \ , \tag{13}$$

where the pecuniary benefits now are given by

$$m^d(B',y) = \vartheta\omega m^r(\omega B',y) + (1-\vartheta)\mathbb{E}_{y'|y}\left[\mathcal{Q}^d(B',y')\right] \ ,$$

and the non-pecuniary ones by

$$\Lambda^d(B',y) = (1-\kappa(1,0)) \times u'\left(\Theta^d\left(B',y\right)\right) \ ,$$

with collateral services

$$\Theta^d\left(B',y\right) = (1-\kappa(1,0)) \times m^d(B',y) \times B' \ .$$

The government's problem can be written recursively. When not in autarky, the government decision problem is given by

$$\mathcal{F}(B,y) = \max_{d\in\{0,1\}} (1-d)\mathcal{F}^r(B,y) + d\mathcal{F}^d(B,y) \, , \tag{14}$$

where the value of repayment $\mathcal{F}^r(B,y)$ satisfies the Bellman equation

$$\mathcal{F}^r(B,y) = \max_{B'} v\left(\tau y + q(B',y)\left(B'-(1-\delta)B\right) - \widetilde{\delta}B\right) + \beta\mathbb{E}_{y'|y}\left[\mathcal{F}(B',y')\right] \ , \tag{15}$$

and the value of default (and autarky) $\mathcal{F}^d(B,y)$ satisfies

$$\mathcal{F}^d(B,y) = v(\tau y) - \phi(\tau y) + \beta\mathbb{E}_{y'|y}\left[\vartheta\mathcal{F}(\omega B,y') + (1-\vartheta)\mathcal{F}^d(B,y')\right] . \tag{16}$$

The equilibrium for the model is defined as follows.

Equilibrium. A Markov-perfect equilibrium consists of value, policy, and bond price functions $\{\mathcal{F}(\cdot)$, $\mathcal{F}^r(\cdot)$, $\mathcal{F}^d(\cdot)$, $\mathcal{B}(\cdot)$, $\mathcal{D}(\cdot)$, $q(\cdot)$, $\mathcal{Q}^r(\cdot)$, $\mathcal{Q}^d(\cdot)\}$, such that

(i) $\mathcal{F}(B,y)$, $\mathcal{F}^r(B,y)$, and $\mathcal{F}^d(B,y)$ satisfy equations (14) to (16),

(ii) $\mathcal{B}(B,y)$ and $\mathcal{D}(B,y)$ solve equations (14) and (15),

(iii) $q(B,y)$, $\mathcal{Q}^r(B,y)$ and $\mathcal{Q}^d(B,y)$ satisfy equations (11) to (13).

Having defined the model equilibrium, it is possible to price a derivative security that will serve as the model equivalent of credit default swaps. A security reflecting the payoff structure of a CDS can be mapped into our model by removing convenience yield from the pricing condition (11):

$$q^{CDS}(B',y) = \frac{1}{1+r^{rf}} \mathbb{E}_{y'|y} \left[\begin{array}{c} (1-\mathcal{D}(B',y')) \left(\widetilde{\delta} + (1-\delta) \mathcal{Q}^{CDS,r}(B',y') \right) \\ +\mathcal{D}(B',y') \mathcal{Q}^{CDS,d}(B',y'), \end{array} \right] \tag{17}$$

To obtain the CDS price that is consistent with equilibrium, the CDS pricing conditions are evaluated at the equilibrium debt policy:

$$\mathcal{Q}^{CDS,r}(B,y) = q^{CDS}\left(\mathcal{B}(B,y),y\right). \tag{18}$$

In the default (and autarky) case, the equilibrium CDS price is given as

$$\mathcal{Q}^{CDS,d}(B,y) = \vartheta \omega \mathcal{Q}^{CDS,r}(\omega B,y) + \frac{1}{1+r^{rf}} \mathbb{E}_{y'|y} \left[(1-\vartheta) \mathcal{Q}^{CDS,d}(B,y') \right]. \tag{19}$$

The crucial difference to the bond pricing expressions represented by equations (12) and (13) is the absence of collateral service: the government bond will trade at a higher price than the CDS price whenever collateral service is positive.

4 Calibration

In this section, we show our model calibration. It is based on Italian data from 2001Q1 to 2012Q4, i.e. one model period corresponds to one quarter. By choosing this truncation point, we exclude periods with unconventional monetary policy measures by the ECB, which are difficult to address in our model framework and the data. The data is compiled from various sources and a complete list can be found in Table B.1. We solve the model numerically using value function iteration over a discretised state space. The computational algorithm is described in detail in Appendix B.2.

4.1 Functional forms and parameter choices

Government. For the income process, we impose a log-normal AR(1) specification:

$$\log y_t = \rho_y \log y_{t-1} + \sigma_y \varepsilon_{y,t}\,, \quad \varepsilon_{y,t} \sim N(0,1)\,. \tag{20}$$

We estimate the income process parameters using Italian GDP data for 1991Q1 to 2012Q4. All data is in real terms, seasonally adjusted, and de-trended using a linear-quadratic trend. Shocks are discretised as proposed by Tauchen (1986). For the period utility function of the government, we assume the specification

$$v(g_t) = \frac{(g_t - \underline{g})^{1-\gamma} - 1}{1-\gamma},$$

with $\underline{g}$ denoting a subsistence level of government spending as in Bocola et al. (2019). We set the government's risk aversion parameter to $\gamma = 2$, which is in line with similar models, such as Hatchondo et al. (2016). To calibrate default risk dynamics, we use the same utility cost function as in Bianchi et al.

(2018):

$$\phi(y) = \max\{d_0 + d_1 \log(y), 0\} .$$

Bond structure. The maturity parameter δ is chosen such that – without default – debt has an average maturity of five years, which corresponds to our empirical specification. Although the average life of Italian bonds slightly exceeds five years during most parts of the sample, using this maturity has several advantages: the benchmark bonds actually issued by the Italian Treasury have a maturity of five years at issuance and CDS contracted over five years are the most commonly traded credit derivative. Fixing the maturity parameter to on-the-run bonds allows us to easily obtain a plausible value for the coupon parameter. We collect coupon rates of all issues of five-year-BTP bonds between 2001 and 2012 from the Italian Department of the Treasury. The (value-weighted) coupon rate was 4.62, which translates to a quarterly coupon rate of 1.15. The probability of a country in default receiving an offer for debt restructuring ϑ and the corresponding recovery rate ω are in the range identified by Cruces and Trebesch (2013).

Investor preferences. Investor utility from collateral services is specified by a CARA function

$$u(\theta_t) = -\frac{\zeta_1}{\zeta_2} \exp\{-\zeta_2(\theta_t - \zeta_3)\} .$$

It follows that the wedge between bond and CDS prices is given by

$$\Lambda_t = (1-\kappa_t)\zeta_1 \exp\{-\zeta_2(1-\kappa_t)m_t B_{t-1}\},$$

which increases in the collateral service weight ζ_1 and decreases in the amount of available collateral $(1-\kappa_t)m_t B_{t-1}$. Keeping haircuts constant, the size of the convenience yield is then governed by ζ_1, while ζ_2 determines the elasticity of the convenience yield with respect to collateral. The real quarterly discount rate of investors r^{rf} is proxied by the average three-month-EURIBOR over quarterly inflation. Consistent with this proxy, we use maturity-matched interest rate swaps r^{swap} as a risk-free reference rate to compute government bond spreads in the data.[15]

Haircut function. For the haircut function $\kappa(\lambda, h)$ we use a simple exponential functional form (see Bindseil, 2014) to capture the negative relationship between the haircut value and default risk,

$$\kappa(\lambda, h) = \begin{cases} \min\{\lambda^{\mu}, \overline{\kappa}\}, & \text{if} \quad h = 1 \\ \overline{\kappa}, & \text{if} \quad h = 0 \, , \end{cases} \tag{21}$$

where λ is the one-period ahead default probability. This functional form implies that haircuts are zero if the bond is default risk-free. The sensitivity parameter is restricted to $\mu \in (0,1]$, i.e. haircuts increase

[15] The EURIBOR itself is not entirely risk-free, since it is based on unsecured lending between AA or higher rated banks. If the European sovereign debt crisis is associated with interbank market stress, this might actually lead to an underestimation of bond spreads $s = y - r^{swap}$ during this time period in the data. However, when taking the spread of EURIBOR against overnight index swaps as a measure of interbank market stress, there is no evidence of systematic underestimation of government bond spreads during the European debt crisis.

Table 1: Parameterisation

Parameter	Value	Source
Government, external parameters		
Autocorrelation ρ_y	0.937	Estimate of (20)
Variance σ_y^2	8.45e-05	Estimate of (20)
Government risk aversion γ	2	Conventional value
Income tax rate τ	1	Normalisation
Financial markets, external parameters		
Investor discount rate r^{rf}	0.0013	Three-month-EURIBOR minus inflation
Coupon parameter χ	0.0115	Average coupon of five-year treasury bonds
Maturity parameter δ	0.05	Five-year average maturity
Recovery rate ω	0.63	Cruces and Trebesch (2013)
Offer probability ϑ	0.08	Cruces and Trebesch (2013)
Haircut elasticity μ	0.4	ECB collateral framework
Maximum haircut $\overline{\kappa}$	0.4	Nyborg (2017)
Calibrated parameters		
Government discount factor β	0.972	Average debt/GDP
Default cost parameter d_0	21.25	Average CDS spread
Default cost parameter d_1	50	Volatility of bond spread
Minimum consumption $\underline{g}$	0.86	Volatility of debt/GDP
Collateral service weight ζ_1	0.4	Average convenience yield
Collateral service CARA coefficient ζ_2	1.5	Volatility of convenience yield

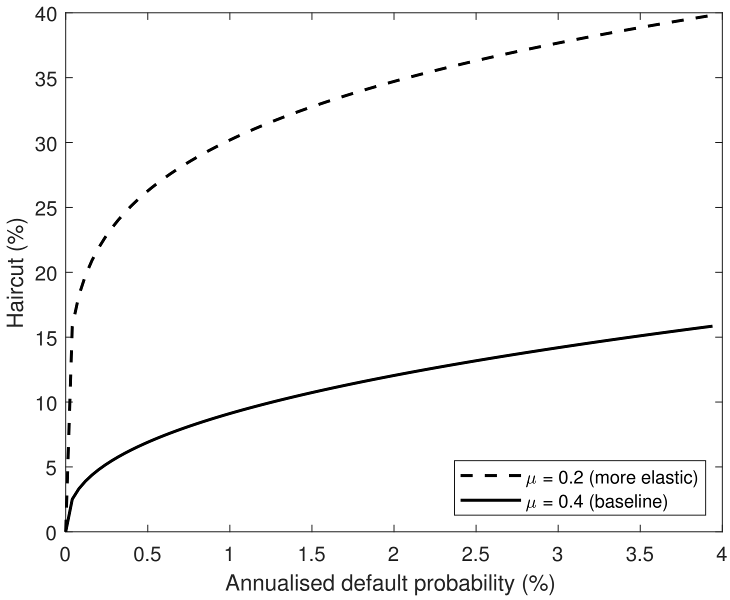
in the default probability, which is in line with collateral frameworks on private markets and of central banks.

For the baseline calibration, we inform μ using ECB haircuts applicable between October 25, 2008 and December 31, 2010 (see Nyborg (2017), Table 5.2). Haircuts in the ECB collateral framework are tied to rating categories and we use the Eurosystem Credit Assessment Framework (ECAF, available under this link) to map ratings into default probabilities. Bonds rated A- or higher correspond to category 2 and have an annualised default probability of up to 0.1%. A bond in category 2 with a residual maturity of five to seven years was subject to a haircut of 3%. For BBB-rated bonds with the same maturity, which fall into credit category 3 with default probabilities from 0.1% to 0.4%, the haircut was 8%.

We assume that bonds rated A- or higher are approximately default-risk free, i.e. haircuts on these bonds account for duration risk, which is not present in our model, and focus on the haircut gap between those two rating categories. The ECB applied a haircut gap of 5% between BBB-rated bonds and bonds rated A- or higher. We compute the difference in default risk between both rating categories by taking the midpoint of the BBB rating bucket and obtain an quarterly default probability of 0.06% for BBB-rated bonds. This yields a value of $\mu = 0.4$, since $(0.06/100)^{0.4} = 0.0523$.[16] Decreasing the elasticity parameter μ implies that haircuts are more responsive to default risk, as demonstrated by the dashed line in Figure 2: while under the baseline calibration, an annualised default probability of 1% implies a haircut of less than 10%, it would be almost 30% for highly elastic haircuts ($\mu = 0.2$). Finally, we apply

[16] Using the midpoint of credit category 2 bucket would yield a quarterly default probability of 0.01% and the haircut implied by $\mu = 0.4$ would be 0.048 instead.

Figure 2: Haircut function



a ceiling to haircuts, which is set to the extraordinary haircut applied to distressed Cypriot and Greek government debt (see Nyborg, 2017, Chapter 5.4). In the case of default or autarky ($h_t = 0$), the haircut is set to $\overline{\kappa}$ as well. In our simulations, the cap is binding in 0.7% of all periods in which the government is not in autarky. Furthermore, as we show in Appendix C.3, our quantitative results regarding the effect of haircut elasticities do not critically depend on $\overline{\kappa}$.

Free parameters. The remaining six parameters $(\beta, d_0, d_1, \underline{g}, \zeta_1, \zeta_2)$ were selected to match key moments of financial market variables observed between 2001Q1 to 2012Q4. The government's discount factor β, subsistence consumption $\underline{g}$, and the utility loss parameters d_0 and d_1 are directly linked to borrowing and default incentives. We set $(\beta, d_0, d_1, \underline{g})$ to jointly target the average and volatility of government debt issuance and default risk as measured by CDS spreads and the public debt-to-GDP ratio, respectively. These mechanics are well-known in quantitative sovereign default models (see Chatterjee and Eyigungor (2012) and Bocola et al. (2019) for a detailed discussion) and turn to the parameters that shape convenience yield and their interaction with default risk. ζ_1 is chosen to match the average convenience yield over the total sample, which was 30 basis points. The elasticity parameter ζ_2 is chosen to match its variance. Since we measure convenience yield as the difference between bond spreads and CDS spreads, we are also targeting the level and volatility of bond spreads by construction. Our calibration is summarised in Table 1.

4.2 Model fit

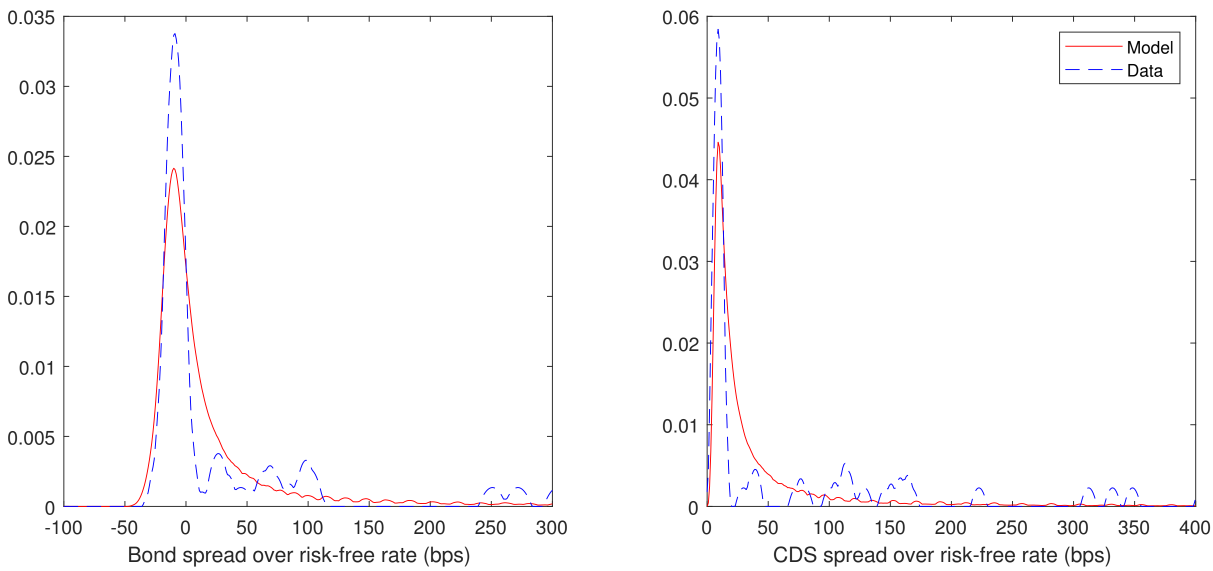
First, we show the levels and volatilities of bond and CDS spreads as well as the debt-to-GDP ratio. We then demonstrate that the model is capable of generating countercyclical debt issuance and finally provide evidence that the interaction of bond supply and convenience yield in the model is consistent with the data.

Table 2: Model fit – financial market variables

Variable	Full sample		Safe episodes		Crisis episodes		Volatility	
	Data	Model	Data	Model	Data	Model	Data	Model
s_t	48	45	-9	-6	139	164	114	144
cds_t	78	76	15	16	192	215	113	176
cy_t	-30	-31	-23	-22	-54	-51	29	33
$\log(Q_t B_t / y_t)$	1.51	1.49	1.46	1.47	1.57	1.59	0.058	0.084

Note: Spreads are annualised and given in basis points, y_t refers to quarterly real GDP. Crisis episodes are all periods with a positive government bond spread. Targeted moments are shown in colour. All model-implied statistics are based on simulations of 50,000 periods, excluding 5,000 burn-in periods. We also exclude all periods in which the government is in financial autarky as well as 40 quarters after reentering financial markets following an exclusion period.

Figure 3: Model fit – distribution of spreads



Financial markets. The results for targeted moments are reported in the left panel of Table 2. The level of debt and all three spreads is matched, whereas the volatility targets show some discrepancy with the data, which is difficult to overcome with our risk-neutral pricing setting. To examine the interaction between convenience yield and credit risk in greater detail, we also report statistics for all periods with a positive bond spread, which we refer to as *crisis periods*. In the data, the bond spread was negative from 2001Q1 until 2008Q4 and turned positive thereafter. As Table 2 shows, the rise in debt levels, bond spreads and CDS spreads associated with the financial crisis is captured by our model. Since our model only features a single aggregate shock as well as risk-neutral bond pricing, the increase in spreads is less pronounced than observed in the data. Notably, the model also captures the decline in the CDS-bond basis, which we interpret as an increase in convenience yield. Lastly, we plot kernel density estimations of bond and CDS spreads obtained from our model against their data counterparts in Figure 3. As in Bocola et al. (2019), both spreads show sizable positive skewness and considerable probability mass at negative bond spreads, consistent with the data.

Debt management. Debt management in developed economies is countercyclical: the government increases borrowing in response to negative fiscal shocks. It should be noted that such behaviour is not present in standard models of sovereign debt and default (see Arellano, 2008). The reason for this is that increases in debt severely lower bond prices in bad times due to the increased risk of default, which typically incentivises the government to reduce its debt issuance. However, as in Bocola et al. (2019), the inclusion of a minimum consumption level makes the government less responsive to debt-elastic interest rate hikes in such instances, such that countercyclical debt and default risk can arise simultaneously. The presence of convenience yield does not change this interaction. The mechanism works as follows. In low-revenue states, bond prices tend to move downward due to increased sovereign risk. Although borrowing is countercyclical, and collateral thus becomes more abundant in principle, the increase in debt outstanding is dominated by default risk, especially because the haircut also grows in this case. The total effect results in a negative comovement between borrowing and bond spreads, which is in line with the data. The model is able to generate sizable negative correlation between government revenues and debt issuance.

Table 3: Model fit – debt management

Variable	Full sample		Crisis episodes	
	Data	Model	Data	Model
$\text{ave}(Q_{t+1}B_t/y_t)$	4.500	4.524	4.804	4.908
$\text{ave}(Q_{t+1}(B_{t+1}-B_t)/y_t)$	0.029	0.008	0.045	0.030
$\text{std}(Q_{t+1}B_t/y_t)$	0.267	0.368	0.113	0.115
$\text{std}(Q_{t+1}(B_{t+1}-B_t)/y_t)$	0.049	0.041	0.054	0.040
$\text{cor}(Q_{t+1}(B_{t+1}-B_t)/y_t, y_t)$	-0.217	-0.772	-0.306	-0.763

Note: Spreads are annualised and given in basis points, y_t refers to quarterly real GDP. Crisis episodes are all periods with a positive government bond spread. All model-implied statistics are based on simulations of 50,000 periods, excluding 5,000 burn-in periods. We also exclude all periods in which the government is in financial autarky as well as 40 quarters after reentering financial markets following an exclusion period.

Convenience yield and government bond supply. Finally, we show that our calibrated elasticity of collateral service implies an empirically plausible correlation between government bond supply and convenience yield. Specifically, we regress convenience yield on the log of outstanding government bonds (relative to GDP). Consistent with our two model extensions presented in Section 6, we control for market illiquidity, measured by bid-ask spreads on five-year Italian government bonds traded on the Milan stock exchange, and for the Eurostoxx volatility index (VSTOXX) to take account of time-varying investor risk aversion.[17] Formally, we run the regression

$$cy_t = \beta_0 + \beta_1 \log\left(\frac{Q_t B_t}{y_t}\right) + \beta \text{Controls}_t + \varepsilon_t \ . \tag{22}$$

The results are shown in Table 4. Despite the small sample size, all four regression paint a conclusive picture of the relationship between convenience yield and government bond supply. The coefficient is

[17] The increase of US Treasury convenience yield during periods of high financial stress is documented by Du et al. (2018).

highly significant after controlling for market illiquidity and investor risk aversion. We run the same regression on the simulated time series implied by our model. Since there is only one persistent exogenous shock, all coefficients are highly significant. Consistent with the data, bond supply is negatively related to convenience yield.

Table 4: Determinants of convenience yield

	Total debt		Long-term debt		Model
Bond supply	-211	-285	-145	-258	-35
	(123)*	(107)***	(80)**	(61)***	(1.93)***
Bid-ask spread	-	0.44	-	0.70	-
	-	(0.62)	-	(0.59)	-
VSTOXX	-	-0.69	-	-1.07	-
	-	(0.54)	-	(0.42)	-

Note: Convenience yield $cy_t \equiv s_t - cds_t$ is given in basis points. The sample period from 2001Q1 to 2012Q3 spans 46 quarters. Newey-West standard errors are shown in paranthesis. *, **, *** indicate significance at the 10%, 5% and 1% levels, respectively.

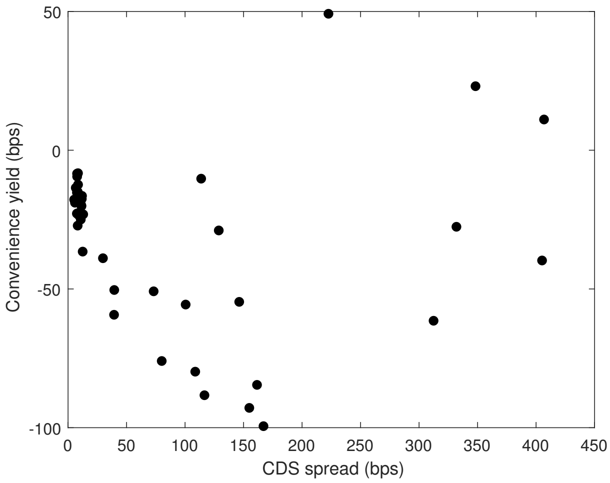
This is consistent with the negative empirical relationship between convenience yield and CDS spreads over our sample. The scatter plot in Figure 4 indicates a negative relationship between sovereign risk as measured by CDS spreads and convenience yield. Together with the regression results in Table 4, we interpret this negative relationship as evidence suggesting a substantial collateral valuation effect. This is in line with the decline of Italian government bond yields below the deposit facility rate in 2008 Corradin et al. (2017), while Italy's debt-to-GDP ratio increased by more than 10 percentage points between 2008Q1 and 2009Q1. Importantly, the negative relationship between debt supply and convenience yield holds after controlling for investor risk aversion. If debt issuance increases the probability of default, the collateral value of outstanding government bonds declines, such that convenience yield increases. The relationship weakens at the height of the sovereign debt crisis, since the haircut component dominates in this case.[18] In the data, this weakened relationship between convenience yield and sovereign risk is also related to the flight-to-quality to German Bunds during the sovereign debt crisis, from which our model abstracts.

5 Model mechanics: A quantitative exploration

Using our calibration, this section examines the impact of convenience yield on financial market variables and public debt management. We proceed in two steps. First, we present a recalibration of the model without convenience yield and compare the dynamics of public debt and default risk to the baseline calibration. These experiments suggest that convenience yield has a quantitatively relevant effect on the debt level, but only small effects on debt and default dynamics. As a second experiment, we separately examine both components of convenience yield: collateral valuation and haircuts. Guided by the analytical results from Section 2, we conduct a comparative statics exercise *(i)* regarding the elasticity

[18] When truncating the sample in 2011Q4 instead of 2012Q4, the correlation coefficient declines to -0.71.

Figure 4: Relationship between convenience yield and default risk



Note: The sample correlation coefficient between cy_t and cds_t is -0.31.

of collateral valuation with respect to the bond supply and *(ii)* regarding the haircut parameter μ. The results of this exercise are in line with the simple analytical framework: more sensitive haircuts reduce the frequency of default if collateral valuation is debt-inelastic and increase the frequency of default if collateral valuation is debt-elastic and thereby provides disciplinary incentives to the government.

5.1 The role of convenience yield

The second column in Table 5 shows that eliminating convenience yield by setting $\zeta_1 = 0$ reduces the government's incentive to issue debt and expose itself to default risk. Therefore, we lower the default cost parameter to $d_0 = 18.13$, such that the average CDS spread matches our target of 78 bps. If d_0 were to remain at its higher baseline value, the government would not find it worthwhile to issue bonds in the risky borrowing region absent convenience yield. Put differently, convenience yield induces higher debt issuance. In contrast to the baseline calibration, the government bond spread mechanically increases up to the level of the CDS spread. This feature stresses our baseline model's ability to reconcile sovereign risk and periods with negative government bond spreads in a unified framework. Compared to the baseline scenario, the debt-to-GDP ratio decreases from 113% to 86% in annualised terms. This is not merely driven by the decline in bond prices induced by the absence of convenience yield. At face value, the debt-to-GDP ratio B_t/y_t is also substantially smaller in this case. The effect on public debt management is quantitatively negligible. The key takeaway from this experiment is that convenience yield increases the debt-to-GDP ratio after controlling for its effect on default risk.

We then recalibrate the model to match the same average debt-to-GDP ratio as in the baseline calibration. This requires the government to be more impatient relative to the baseline case. To counter the positive impact of higher impatience on the incentive to default, the recalibrated default cost parameter needs to be increased to match our target. The resulting adjustment requires setting $\beta = 0.962$ and $d_0 = 25.0781$. The results are displayed in the third column of Table 5. Compared to the baseline calibration and the case without convenience yield, the effect of convenience yield on financial market variables and government debt management is relatively small. These results suggest that convenience yield plays a quantitatively relevant role in reconciling high debt-to-GDP ratios. Finally, Panel A of Table 5 highlights another important feature of our model that standard sovereign default models cannot address by construction. In more than half of all periods in which the government is not in autarky, the bond spread is negative.

5.2 The role of collateral scarcity

In a second step, we evaluate the relative importance of collateral scarcity. We recalibrate the model under the assumption of a constant marginal utility of collateral service, such that collateral valuation is given by

$$\Lambda_t = (1-\kappa(\lambda_t)) \times \zeta_1 \,.$$

It should be noted that convenience yield still depends negatively on bond supply through the haircut function and default risk. We again recalibrate the model ($d_0 = 23.3$ and $\zeta_1 = 0.0007$) to ensure that average CDS and government bond spreads remain consistent with the data moments. As the fourth column of Table 5 reveals, the debt-to-GDP ratio declines to 89% (in annualised terms), which is only slightly higher than in the case without convenience yield. Furthermore, the discount factor has to be reduced to $\beta = 0.963$ to match the debt-to-GDP ratio of the baseline calibration, suggesting that collateral scarcity is a crucial component of convenience yield. As the last two columns of Table 5 show, collateral scarcity also enables the model to fit the data more closely. Once collateral scarcity is set to zero, the volatility of convenience yield (4 bps) falls almost one order of magnitude short of its data moment (29 bps).

5.3 The role of haircuts

Up to this point, we have discussed how convenience yield improves the quantitative sovereign default model's ability to match the level and joint dynamics of government bond supply, default risk, and bond spreads. In this section, we show how the haircut component of convenience yield affects sovereign risk and how it interacts with collateral scarcity. The left-hand panel of Table 6 displays the results for the baseline specification of $u(\theta)$.

The upper panel of Table 6 shows that both the level and volatility of spreads decrease with μ in the baseline case with scarcity. The reason for this observation lies in the responsiveness of the collateral valuation component to the supply of government bonds. If a highly elastic collateral valuation is combined with a highly elastic haircut schedule, the collateral scarcity effect dominates the haircut effect and renders higher debt issuance and default risk optimal to the government. Making haircuts less responsive to default risk by increasing μ softens this effect.

The impact of μ on the level and volatility of spreads is reversed in the case of a debt-inelastic collateral valuation. Here, less elastic haircut schedules (with a higher μ) are associated with higher default rates and spreads, since the disciplinary effects of collateral scarcity is absent. Panel B of Table 6 reveals that haircut elasticities also affect the conduct of public debt management. In the case without scarcity, a less elastic convenience yield implies that bond prices are less responsive to a negative revenue shock, holding average debt constant. Consequently, fiscal policy is more countercyclical. The same

Table 5: Convenience yield – selected moments

Variable	Baseline ($\beta = 0.972$)	No CY	No CY ($\beta = 0.962$)	Const. CY	Const. CY ($\beta = 0.963$)
Panel A: Financial markets					
ave(s_t)	45	78	79	47	48
ave(cds_t)	76	78	79	76	78
ave(cy_t)	-31	0	0	-29	-30
std(s_t)	144	172	181	171	172
std(cds_t)	176	172	181	175	175
std(cy_t)	33	0	0	4	4
Negative spread (%)	49.6	0	0	59.6	58.3
Panel B: Debt management					
ave(B_t/y_t)	4.023	3.081	4.007	3.190	3.989
ave($Q_{t+1}B_t/y_t$)	4.524	3.431	4.443	3.584	4.485
ave($Q_{t+1}(B_{t+1}-B_t)/y_t$)	0.008	0.007	0.007	0.007	0.007
std($Q_{t+1}B_t/y_t$)	0.368	0.323	0.303	0.320	0.296
std($Q_{t+1}(B_{t+1}-B_t)/y_t$)	0.041	0.039	0.037	0.039	0.037
cor($Q_{t+1}(B_{t+1}-B_t)/y_t, y_t$)	-0.772	-0.762	-0.718	-0.744	-0.705

Note: For all experiments, d_0 is recalibrated to match average CDS spreads. Spreads are annualised and given in basis points, y_t refers to quarterly real GDP. All model-implied statistics are based on simulations of 50,000 periods, excluding 5,000 burn-in periods. We also exclude all periods in which the government is in financial autarky as well as 40 quarters after reentering financial markets following an exclusion period. The default rate (%) and the negative spread (%) are computed relative to the number of periods in which the government has access to financial markets.

argument applies to the case with scarcity. However, the effects on public debt management are rather small in magnitude.

Taken together, our experiments indicate that the composition of effective convenience yield matters for sovereign risk and the conduct of fiscal policy. Altering the responsiveness of haircut schedules can change the level and volatility of CDS spreads by up to 19 basis points and default rates by up to 0.1 percentage point. These effects are quantitatively relevant, given that average government bond spreads are around 50 basis points and the default rate around 1.5% in our baseline calibration. The results furthermore suggest that the design of haircut schedules (either in public or private market segments) could benefit from taking account of the nature of collateral demand.

6 Model extensions

This section presents two extensions of the baseline model shown in Section 3. First, we augment the model to include trading frictions to show that our results are not driven by market illiquidity as a latent factor. Second, we allow for a negative correlation between collateral demand and government revenues. This implies that convenience yield is particularly high in times of crisis for a different reason than collateral scarcity.

Table 6: Haircut and collateral valuation components – selected moments

	With scarcity			*Without scarcity*		
Panel A: Financial markets	$\mu = 0.1$	$\mu = 0.4$	$\mu = 0.7$	$\mu = 0.1$	$\mu = 0.4$	$\mu = 0.7$
ave(s_t)	48	45	41	48	48	49
ave(cds_t)	81	76	62	77	78	78
ave(cy_t)	-33	-31	-31	-29	-30	-29
std(s_t)	145	144	137	171	172	172
std(cds_t)	179	176	167	175	175	176
std(cy_t)	35	33	31	4	4	4
Default rate (%)	1.36	1.33	1.24	1.33	1.38	1.38
Panel B: Debt management						
ave(B_t/y_t)	4.055	4.023	4.011	3.986	3.989	3.990
ave$(Q_{t+1}B_t/y_t)$	4.552	4.524	4.518	4.482	4.485	4.485
ave$(Q_{t+1}(B_{t+1}-B_t)/y_t)$	0.008	0.008	0.007	0.007	0.007	0.007
std$(Q_{t+1}B_t/y_t)$	0.369	0.368	0.366	0.298	0.296	0.296
std$(Q_{t+1}(B_{t+1}-B_t)/y_t)$	0.042	0.041	0.041	0.037	0.037	0.037
cor$(Q_{t+1}(B_{t+1}-B_t)/y_t, y_t)$	-0.781	-0.772	-0.771	-0.703	-0.705	-0.705

Note: Effects of varying the haircut parameter μ. A low μ corresponds to a highly elastic haircut schedule (see also Figure 2). Spreads are annualised and given in basis points, y_t refers to quarterly real GDP. All model-implied statistics are based on simulations of 50,000 periods, excluding 5,000 burn-in periods. We also exclude all periods in which the government is in financial autarky as well as 40 quarters after reentering financial markets following an exclusion period. The default rate (%) and negative spread (%) are computed relative to the number of periods in which the government has access to financial markets. Default rate (%) is computed relative to the number of periods where the government has access to financial markets.

Market illiquidity. In this model version, each period is divided into two sub-periods. In the first sub-period, investors are subject to uninsurable idiosyncratic shocks to the valuation of collateral services. However, we assume that the government bond market is not open at this stage and investors can only trade bonds with dealers on a decentralised over-the-counter market.[19] Restricting their ability to trade with each other will give rise to endogenous bid-ask spreads. Tax revenues realise at the beginning of the second sub-period. All agents consume in the second sub-period, when the competitive and centralised government bond market is open.

We assume that investors are subject to i.i.d. preference shocks. With a probability of $\frac{1}{2}$, investors are either a high-collateral-valuation or a low-collateral-valuation type investor, which we denote by $i \in \{L, H\}$. Specifically, investor types differ in the utility derived from a bond's collateral services, with $u_L(\cdot) < u_H(\cdot)$. Investors cannot directly trade bonds amongst each other or with the government in the first sub-period. They can, however, adjust their bond holdings by trading with dealers in exchange for an endogenously determined fee $\phi_{i,t}$. Dealers have access to a competitive inter-dealer market and intermediate between buyers and sellers as in Lagos and Rocheteau (2009). The terms of trade are determined bilaterally between a dealer and an investor via Nash bargaining. The bargaining power of both sides is normalised to $\frac{1}{2}$ in the following. We assume that there are no search frictions, such that

[19] Dividing a period into two sub-periods that represent a decentralised and a centralised market, respectively, is in the spirit of Lagos and Wright (2005).

each buyer/seller is matched to a dealer. In Appendix A, we derive the bond pricing condition consistent with investors' optimality conditions:

$$q_t = \frac{1}{1+r^{rf}} \times \underbrace{m_{t+1}}_{\text{Payoff}} \times \underbrace{(1+\Lambda_{t+1})}_{\text{Convenience yield}} , \tag{23}$$

with

$$\Lambda_{t+1} \equiv \underbrace{(1-\kappa_{t+1})}_{1-\text{Haircut}} \times \underbrace{\mathbb{E}_i \left[u_i'(\theta_{t+1}) + \frac{1}{2} \times \left\{ u_i'(\widetilde{\theta}_{i,t+1}) - u_i'(\theta_{t+1}) \right\} \right]}_{\text{Collateral valuation}} . \tag{24}$$

While the first-order condition does not change relative to the baseline version, non-pecuniary benefits are affected by trading frictions when dealers have bargaining power vis-à-vis investors. Specifically, the term $u_i'(\widetilde{\theta}_{i,t+1}) - u_i'(\theta_{t+1})$ measures the net marginal gains from trading, with $\widetilde{\theta}_{i,t+1}$ denoting the collateral value of the bond position held by an investor of type i after trading on the decentralised market (see Appendix A for details). Without preference heterogeneity, (24) collapses to the baseline expression (10).

Since market illiquidity is tied to convenience yield in our model, the CDS price also does not contain illiquidity discounts. To disentangle convenience yield and market illiquidity, we can, however, exploit the analytical tractability of the decentralised/centralised market framework. As outlined in Appendix A, the bid-ask spread as a measure of illiquidity is given by

$$ba^h(B,y) = \frac{1}{2}\left[\frac{u_H(\Theta_H^h(B,y,\widetilde{b}_H(B))) - u_H(\Theta^h(B,y,B))}{\widetilde{b}_H(B) - B} - \frac{u_L(\Theta_L^h(B,y,\widetilde{b}_L(B))) - u_L(\Theta^h(B,y,B))}{\widetilde{b}_L(B) - B} \right] ,$$

which in turn depends on the credit status $h \in \{0,1\}$. To quantify the effects of market illiquidity, we extend the collateral valuation function by two parameters that enable us to match both the level and volatility of bid-ask spreads (see Appendix C.2). The high-valuation type investor will always assign a higher value to collateral services. We add the parameter ζ_3 to control the trading motive between high-valuation and low-valuation type investors. In Table C.2, we demonstrate that the model fit in this extension is very similar to the baseline calibration.

Time-varying collateral demand. In the baseline model, we have implicitly assumed that adverse shocks to fiscal conditions are uncorrelated with investors' collateral demand. This simplification is conceptually reasonable under the assumption that investors are (mostly) foreign financial institutions and that sovereign risk is driven exclusively by fiscal shocks that are orthogonal to investors' financial conditions. However, since the sovereign debt crisis was associated with financial stress, this simplification can be called into question. Financial institutions might demand more collateral during a sovereign debt crisis to borrow on the secured interbank market segment or to participate in lender-of-last-resort policies by central banks (Drechsler et al., 2016).[20] This effectively increases the non-pecuniary benefits from government bond holdings during times of crisis. Formally, this corresponds to making collateral

[20]The positive relationship between liquidity premia and financial stress is discussed in Vayanos (2004). See also Corradin et al. (2017) and the references therein.

Table 7: The effect of haircuts in different model extensions

	Illiquidity			*Time-varying CY*		
	$\mu = 0.1$	$\mu = 0.4$	$\mu = 0.7$	$\mu = 0.1$	$\mu = 0.4$	$\mu = 0.7$
ave(s_t)	48	45	41	49	49	48
ave(cds_t)	99	78	65	78	78	76
Default rate (%)	1.36	1.33	1.24	1.33	1.36	1.34
ave(B_t/y_t)	4.06	4.02	4.01	3.90	3.88	3.87
ave(q_tB_t/y_t)	4.56	4.48	4.47	4.38	4.35	4.35

Note: Effects of varying the haircut parameter μ. A low μ corresponds to a highly elastic haircut schedule (see also Figure 2). Spreads are annualised and given in basis points, y_t refers to quarterly real GDP. All model-implied statistics are based on simulations of 50,000 periods, excluding 5,000 burn-in periods. We also exclude all periods in which the government is in financial autarky as well as 40 quarters after reentering financial markets following an exclusion period.

demand dependent on the exogenous revenue state, $u(\theta_t, y_t)$, such that $u'(\theta_t, y_t^{low}) > u'(\theta_t, y_t^{high})$. We show that this model extension can enhance the model's ability to replicate the empirical relationship between bond supply and convenience yield, but leaves our key comparative statics exercise concerning haircut elasticities intact. Again, the specification of functional forms and the recalibration is presented in Appendix C.2, while the model fit is shown in Table C.3.

Results. Table 7 summarises our main comparative statics experiment with respect to haircut elasticities for both extensions. The left-hand panel considers the case with market illiquidity and only shows very small differences compared to our main results (Table 6). Likewise, the model with time-varying convenience yield (and a slightly less elastic collateral valuation function) delivers the same comparative statics as our baseline model.

7 Conclusion

In this paper, we have studied how convenience yield, sovereign default risk, and the supply of government bonds interact through the lenses of a macroeconomic model. First, we have shown in an analytically tractable model version that convenience yield non-trivially interacts with sovereign risk and the supply of government bonds. Second, to investigate the quantitative relevance of our analytical results, we considered a more elaborate model version that was calibrated to Italy. Despite its parsimonious structure, our model can generate the main observed properties of sovereign debt, credit risk, and sovereign bond spreads. To understand the role of convenience yield in the presence of default risk, we provide a decomposition of convenience yield into individual components. Our analysis suggests that the elasticity of collateral valuation and haircut schedules applied to collateral with respect to the supply of government bonds can have sizable effects on sovereign debt and default.

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A Market illiquidity: Investor problem

This section presents the investor problem with trading frictions and the derivation of the market illiquidity-augmented bond-pricing condition (23). Before describing the investors' maximisation problem, Figure A.1 summarises the within-period timing structure.

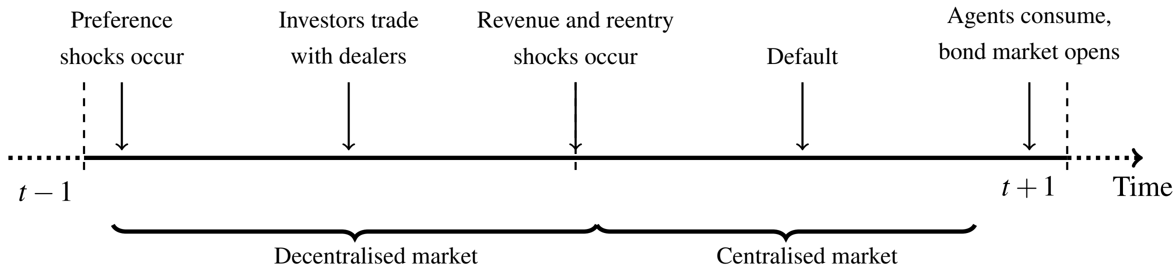

Figure A.1: Within-period timing

Maximisation problem. The investor-decision problem is formulated recursively. An investor's value function in the second sub-period (centralised market) is given as

$$\mathcal{W}_t(\widetilde{b}_t, a_t) = \max_{b_t, c_t} \left\{ c_t + \frac{1}{1+r^{rf}} \mathbb{E}_t \left[\mathcal{V}_{t+1}(b_t) \right] \right\}$$

subject to the budget constraint

$$c_t = e - a_t + k_t \widetilde{b}_t - q_t b_t,$$

where $\widetilde{b}_t$ are bond holdings at the beginning of the second sub-period and a_t are payments owed to a dealer from trading in the decentralised market in the first sub-period of period t (described below).

The continuation value in the second sub-period reflects the occurrence of idiosyncratic preference shocks. The probability of becoming an L-type investor is normalised to $\frac{1}{2}$. An investor's value function at the beginning of a period, before idiosyncratic preference shocks materialise, is given by

$$\mathcal{V}_t(b_{t-1}) = \frac{1}{2} \mathcal{V}_{L,t}(b_{t-1}) + \frac{1}{2} \mathcal{V}_{H,t}(b_{t-1}),$$

with the value of being an i-type investor given by

$$\begin{aligned} \mathcal{V}_{i,t}(b_{t-1}) =& u_i\big((1-\kappa_t) m_t \widetilde{b}_{i,t}\big) + \mathbb{E}_{t-1}\left[\mathcal{W}_t\big(\widetilde{b}_{i,t}, -\widetilde{q}_t(\widetilde{b}_{i,t} - b_{t-1}) - \phi_{i,t}\big) \right] \\ =& u_i\big((1-\kappa_t) m_t \widetilde{b}_{i,t}\big) + m_t(\widetilde{b}_{i,t} - b_t) - \widetilde{q}_t(\widetilde{b}_{i,t} - b_{t-1}) - \phi_{i,t} + \mathbb{E}_{t-1}\left[\mathcal{W}_t(b_t, 0) \right], \end{aligned}$$

which holds for an arbitrary b_t. Here, $\phi_{i,t}$ is a fee charged by dealers and $\widetilde{q}_t$ is the competitive bond price on the inter-dealer market (both described below). It should be noted that the second equality is due to the linearity of the value function $\mathcal{W}_t(\cdot)$ in $\widetilde{b}_t$ and a_t.

Decentralised market. To adjust their bond holdings in response to the i shock, investors contact dealers. The terms of trade between dealers and investors are determined bilaterally via Nash bargaining. For an i-type investor, the bargaining threat point is

$$\overline{\mathcal{V}}_{i,t}(b_{t-1}) = u_i\big((1-\kappa_t)m_t b_{t-1}\big) + \mathbb{E}_{t-1}\left[\mathcal{W}_t(b_{t-1},0)\right],$$

such that the surplus from trading is given by the utility gain from trading $u_i(\widetilde{\theta}_{i,t}) - u_i\big((1-\kappa_t)m_t b_{t-1}\big)$, net of pecuniary benefits, and the fee payed to dealers

$$S_{i,t}(b_{t-1}) = u_i(\widetilde{\theta}_{i,t}) - u_i\big((1-\kappa_t)m_t b_{t-1}\big) + m_t(\widetilde{b}_{i,t} - b_{t-1}) - \widetilde{q}_t(\widetilde{b}_{i,t} - b_{t-1}) - \phi_{i,t}.$$

In the expression for the surplus, $\widetilde{\theta}_{i,t} \equiv (1-\kappa_t)m_t\widetilde{b}_{i,t}$ is the (haircut-adjusted) value of bond holdings after adjustment for the bond position and $(\widetilde{b}_{i,t}, \phi_{i,t})$ constitute the terms of trade, which consist of the investor's bond holdings after the meeting $\widetilde{b}_{i,t}$ and the fee charged by the dealer $\phi_{i,t}$. Payments owed to dealers consist of the fee and the desired adjustment of the bond positions,

$$a_{i,t} = \phi_{i,t} + \widetilde{q}_t(\widetilde{b}_{i,t} - b_{t-1}),$$

and are settled in the subsequent centralised market. A dealer's surplus in an i-type meeting simply equals $\phi_{i,t}$, which is consumed by the dealer in the centralised market. Dealers do not acquire bonds in the centralised market. The investors' bargaining power is α, the terms of trade solve the generalised Nash bargaining problem

$$\max_{\widetilde{b}_{i,t},\phi_{i,t}} \left[u_i\big((1-\kappa_t)m_t\widetilde{b}_{i,t}\big) - u_i\big((1-\kappa_t)m_t b_{t-1}\big) + m_t(\widetilde{b}_{i,t} - b_{t-1}) - \widetilde{q}_t(\widetilde{b}_{i,t} - b_{t-1}) - \phi_{i,t}\right]^{\alpha} \phi_{i,t}^{1-\alpha},$$

which leads to the two first-order conditions

$$\widetilde{q}_t = (1-\kappa_t)m_t u_i'(\theta_{i,t}) + m_t, \tag{A.1}$$

$$\phi_{i,t} = (1-\alpha)\Big(u_i(\widetilde{\theta}_{i,t}) - u_i(\theta_t) + m_t(\widetilde{b}_{i,t} - b_{t-1}) - \widetilde{q}_t(\widetilde{b}_{i,t} - b_{t-1})\Big). \tag{A.2}$$

It should be noted that the dealer fee simply equals the dealer's bargaining power $1-\alpha$ multiplied by the total surplus $S_{i,t}(b_{t-1}) + \phi_{i,t}$. As in Lagos and Rocheteau (2009), the dealer fee can be expressed in terms of a meeting-specific bond price $\widetilde{q}_{i,t}$:

$$\phi_{i,t} = (\widetilde{q}_{i,t} - \widetilde{q}_t)(\widetilde{b}_{i,t} - b_{t-1}).$$

Using this relationship, the meeting-specific bond price can be expressed as a weighted average of the inter-dealer market price $\widetilde{q}_t$ and the total surplus net of payments $\widetilde{q}_{i,t}(\widetilde{b}_{i,t} - b_{t-1})$ divided by the net trading position:

$$\widetilde{q}_{i,t} = \alpha\widetilde{q}_t + (1-\alpha)\frac{u_i(\widetilde{\theta}_{i,t}) - u_i(\theta_t) + m_t(\widetilde{b}_{i,t} - b_{t-1})}{\widetilde{b}_{i,t} - b_{t-1}}.$$

This will be useful for deriving bid-ask spreads below. If the investor holds all bargaining power ($\alpha = 1$), the dealer does not charge a mark-up/mark-down, such that $\widetilde{q}_{i,t} = \widetilde{q}_t$. If the investor is a net buyer ($\widetilde{b}_{i,t} > b_{t-1}$), then the ask price of the dealer is $\widetilde{q}_{i,t}$, which exceeds the inter-dealer price $\widetilde{q}_t$ whenever $\alpha < 1$. Similarly, if the investor is a net seller, $\widetilde{q}_{i,t}$ is the bid price, which is below $\widetilde{q}_t$ for $\alpha < 1$. Consistent with the quantitative analysis, we normalise the bargaining power to $\alpha = \frac{1}{2}$ in the following. A commonly used measure of the extent to which a market is affected by trading frictions is the bid-ask spread, which in our model is given as

$$\widetilde{q}_{H,t} - \widetilde{q}_{L,t} = \frac{\phi_{H,t}}{\widetilde{b}_{H,t} - b_{t-1}} - \frac{\phi_{L,t}}{\widetilde{b}_{L,t} - b_{t-1}} \,. \tag{A.3}$$

Market clearing in the competitive inter-dealer market requires

$$\frac{1}{2}\widetilde{b}_{L,t} + \frac{1}{2}\widetilde{b}_{H,t} = B_{t-1},$$

with government bond supply B_{t-1}, such that

$$u_L'\big((1-\kappa_t)m_t\widetilde{b}_{L,t}\big) = u_H'\Big((1-\kappa_t)m_t \times 2 \times \Big(B_{t-1} - \frac{1}{2}\widetilde{b}_{L,t}\Big)\Big),$$

determines $\widetilde{b}_{L,t}$ and $\widetilde{b}_{H,t}$ via market clearing.

Centralised market. For the centralised market, the investor first-order condition is

$$-q_t + \frac{1}{1+r^{rf}} \frac{d\mathbb{E}_t\left[\mathcal{V}_{t+1}(b_t)\right]}{db_t} = 0. \tag{A.4}$$

The value $\mathcal{V}_{t+1}(\cdot)$ can be written as

$$\mathcal{V}_{t+1}(b_t) = \mathbb{E}_i\left[u_i(\theta_t) + S_{i,t+1}(b_t)\right] + \mathbb{E}_t\left[\mathcal{W}_{t+1}(b_t,0)\right],$$

such that

$$\begin{aligned}
\frac{d\mathcal{V}_{t+1}(b_t)}{db_t} &= \mathbb{E}_i\left[(1-\kappa_{t+1})m_{t+1}u_i'(\theta_{t+1}) + \frac{dS_{i,t+1}(b_t)}{db_t}\right] + \frac{\partial\mathbb{E}_t\left[\mathcal{W}_{t+1}(b_t,0)\right]}{\partial b_t} \\
&= \mathbb{E}_i\left[(1-\kappa_{t+1})m_{t+1}u_i'(\theta_{t+1}) + \frac{dS_{i,t+1}(b_t)}{db_t}\right] + m_{t+1} \,.
\end{aligned}$$

The effect of individual bond holdings on trading frictions is given by

$$\begin{aligned}
\frac{dS_{i,t+1}(b_t)}{db_t} =& (1-\kappa_{t+1})m_{t+1}\frac{d\widetilde{b}_i(b_t)}{db_t}u_i'(\widetilde{\theta}_{i,t+1}) - (1-\kappa_{t+1})m_{t+1}u_i'(\theta_{t+1}) \\
&+ m_{t+1}\left(\frac{d\widetilde{b}_i(b_t)}{db_t} - 1\right) - \widetilde{q}_{t+1}\left(\frac{d\widetilde{b}_i(b_t)}{db_t} - 1\right) - \frac{d\phi_i(b_t)}{db_t}.
\end{aligned}$$

Using

$$\frac{d\phi_i(b_t)}{db_t} = \frac{1}{2}\left[\begin{array}{c}(1-\kappa_{t+1})m_{t+1}\frac{d\widetilde{b}_i(b_t)}{db_t}u_i'(\widetilde{\theta}_{i,t+1})-(1-\kappa_{t+1})m_{t+1}u_i'(\theta_{t+1})\\ +m_{t+1}\left(\frac{db_i(b_t)}{db_t}-1\right)-\widetilde{q}_{t+1}\left(d\frac{\widetilde{b}_i(b_t)}{db_t}-1\right)\end{array}\right],$$

together with (A.1), this derivative can be written as

$$\frac{dS_{i,t+1}(b_t)}{db_t} = \frac{1}{2}\left[u_i'(\widetilde{\theta}_{i,t+1})-u_i'(\theta_{t+1})\right](1-\kappa_{t+1})m_{t+1},$$

such that

$$\frac{d\mathcal{V}_{t+1}(b_t)}{db_t} = \mathbb{E}_i\left[u_i'(\theta_{t+1})+\frac{1}{2}\times\left\{u_i'(\widetilde{\theta}_{i,t+1})-u_i'(\theta_{t+1})\right\}\right](1-\kappa_{t+1})m_{t+1}+m_{t+1}.$$

Combining this condition with the investor first-order condition (A.4) then yields the bond-pricing equation (23). As in Lagos and Wright (2005), the linearity of investor preferences with respect to consumption and idiosyncratic preference shocks being i.i.d. implies that investors choose the same bond holdings b_t in the centralised market regardless of their current holdings $\widetilde{b}_{i,t-1}$.

B Numerical appendix

B.1 Data sources

This section contains the data sources used in the quantitative analysis.

Debt. Debt service is taken from monthly reports of the Italian Department of the Treasury. To focus on debt with appropriate maturities, we focus on redemption and coupon payments of bonds with maturities of at least one year. The inclusion of short-term liabilities (Treasury Bills, known as BOT) would introduce a relatively large amount of outstanding debt that is potentially rolled over multiple times each period. Coupon data are obtained from annual reports on debt issuance also published by the Treasury.

Income process. The GDP data for Italy is obtained from the St. Louis Fed database, going from 1961Q1 to 2012Q4. The data is in real terms and seasonally adjusted. The data is logged and de-trended using deviations from a linear-quadratic trend. We subsequently impose an AR(1) structure and obtain $(\rho_y, \sigma_y^2) = (0.937, 8.45e-05)$.

Investors. As a proxy of the discount rate of investors, we use the three-month-EURIBOR data (1991Q1 to 2012Q4) from the Bundesbank and (quarterly) euro area inflation rates. The real rate is simply obtained by subtracting quarterly inflation from the three-month EURIBOR. The data on German bonds used to construct the maximum convenience yield are taken from Datastream.

B.2 Solution algorithm

This section presents details on the numerical solution algorithm used to solve the government problem.

Taste shocks. We solve the model numerically using value function iteration on a discretise state space without interpolation. The debt grid is denoted by $\mathfrak{B}$. The expected continuation value is defined as $F(B', y) \equiv \mathbb{E}_{y'|y}\mathcal{F}(B', y')$. As already pointed out by Chatterjee and Eyigungor (2012), convergence problems typically arise in this class of models. We follow Gordon (2019) by introducing Gumbel-distributed taste shocks z to randomise over debt and default choices:

$$\mathcal{F}^r(B, y', z) = \max_{B'} \left\{ v\left(\tau y + q\left(B', y\right)\left(B' - (1-\delta)B\right) - \widetilde{\delta} B\right) + \widetilde{\beta} F(B', y) + \sigma_z z^{B'} \right\} .$$

Throughout the algorithm we standardise the value function prior to computing choice probabilities and expected choices. The standardisation is immediately cancelled out when choice probabilities are computed. For the expected value of the maximum choice, consider the well-known expression

$$\mathbb{E}\left(\max_j V(j) + \sigma \varepsilon_j\right) = \sigma \log\left(\sum_j \exp\left\{\frac{V(j)}{\sigma}\right\}\right) . \tag{B.1}$$

Table B.1: Data sources and ticker

Series	Source	Mnemonic	Frequency
Redemption yield, five-year benchmark BTP	Bank of Italy	MFN_BMK.M.020.922.0.EUR.205	Monthly average
Turnover, five-year BTP	Bank of Italy	MFN_QMTS.D.020.926.MKV.EUR.9	Monthly average
Total debt	Bank of Italy	FPI_FP.M-IT.S13.MGD.SBI3.101.112.FAV.EUR.EDP	Monthly
Net issuance, medium-term and long-term	Bank of Italy	FPI_FP.M.IT.S13.F32.SBI3.103.115.COV.EUR.FPBI	Monthly
Debt service, medium-term and long-term	Department of the Treasury	Monthly reports	Monthly
Turnover, total debt	Bank of Italy	MFN_QMTS.D.100010.926.MKV.EUR.9	Monthly average
Bid-ask spreads, five-year BTP	Department of the Treasury	Quarterly bulletins	Monthly
EURIBOR swap, five-year	Datastream	ICEIB5Y	Daily
CDS spread, Italy, five-year	Bocola (2016)		Daily
Three-month EURIBOR	Bundesbank	BBK01.SU0316	Daily
Euro area CPI	St. Louis Fed	EA19CPALTT01IXOBQ	Quarterly
Redemption yield, Germany, five-year	Datastream	TRBD5YT	Daily
GDP, Italy	St. Louis Fed	LORSGPORITQ661S	Quarterly

where j denotes the choice in the next period. Dependence on current persistent states is omitted. Expanding both sides by an arbitrary constant $\overline{V}$ yields

$$\overline{V}+\mathbb{E}\left(\max_j V(j)-\overline{V}+\sigma\varepsilon_j\right)=\sigma\log\left(\sum_j\exp\left\{\frac{V(j)-\overline{V}}{\sigma}\right\}\right). \tag{B.2}$$

The right-hand side of (B.2) can be rearranged as

$$\begin{aligned}\overline{V}+\sigma\log\left(\sum_j\exp\left\{\frac{V(j)-\overline{V}}{\sigma}\right\}\right)&=\overline{V}+\sigma\log\left(\exp\left\{-\frac{-\overline{V}}{\sigma}\right\}\sum_j\exp\left\{\frac{V(j)}{\sigma}\right\}\right)\\&=\overline{V}+\sigma\log(\exp\left\{-\frac{\overline{V}}{\sigma}\right\})+\sigma\log\left(\sum_j\exp\left\{\frac{V(j)}{\sigma}\right\}\right)\\&=\sigma\log\left(\sum_j\exp\left\{\frac{V(j)}{\sigma}\right\}\right).\end{aligned}$$

Plugging back into (B.2) gives

$$\mathbb{E}\left(\max_j V(j)+\sigma\varepsilon_j\right)=\overline{V}+\sigma\log\left(\sum_j\exp\left\{\frac{V(j)-\overline{V}}{\sigma}\right\}\right). \tag{B.3}$$

When $\overline{V}=\max_j V(j)$ is chosen, each term in the exponent on the left-hand side is negative and the problem remains numerically tractable. To speed up computation, we use a version of the divide-and-conquer algorithm proposed by Gordon and Qiu (2018) and Gordon (2019) that truncates numerically irrelevant choices based on policy function monotonicity. The improved convergence properties compared to a brute-force algorithm are particularly relevant for the large state space required by our model. Since there are some (numerically small) non-monotonicities at some points in the state space, we use a guess-and-verify approach. We compute the equilibrium using the divide-and-conquer approach until convergence. After the final step, we use that solution to compute another iteration. We consider the solution obtained using the divide-and-conquer approach appropriate if the equilibrium objects remain reasonably near to the solution from the previous iteration.

Algorithm. To start off the algorithm, we use the final period of a finite-horizon model to determine default states for iteration 0. Given the results of iteration ι (or initialisation), each iteration $\iota+1$ proceeds as follows:

1. For all states (B,y'), evaluate debt choices B' according to

$$\left\{\overline{B},0,\frac{1}{2}\overline{B},\frac{3}{4}\overline{B},\frac{1}{4}\overline{B},\frac{7}{8}\overline{B},\frac{5}{8}\overline{B},\dots\right\}.$$

   (a) Determine the value of default:

$$\mathcal{F}^d_{\iota+1}(B,y')=v\left(\tau y'-\phi(\tau y')\right)-\phi(\tau y')+F^d_\iota(B,y').$$

   (b) Evaluate every numerically relevant debt choice in $\mathfrak{B}$, given current debt over the interval

$[B'_-, B'_+]$ (elements of the grid $\mathfrak{B}$) with

$$B'_- = \min\Big(B' | P(\mathcal{B}(B_-)) > \varepsilon\Big), \quad \text{and} \quad B'_+ \max\Big(B' | P(\mathcal{B}(B_+)) > \varepsilon\Big) ,$$

where B_+ and B_- respectively denote the next largest and smallest current debt stock that has already been evaluated. At the upper bound, the choices are not bounded, while for $\underline{B} = 0$, they are only bounded from above. The value of choosing $B^j \in \mathfrak{B} | B_- < B^j < B_+$ is given by

$$\mathcal{F}^r_{\iota+1}(B^j|B,y') = v\Big(\tau y' + \mathcal{Q}^r_\iota(B^j,y')\left(B^j - (1-\delta)B\right) - \widetilde{\delta}B\Big) + \widetilde{\beta}F_\iota(B^j,y') .$$

(c) Compute debt choice and default probabilities using the Type I extreme value distribution:

$$\Pr(d=1|B,y') = \frac{\exp\left(\mathcal{F}^d_{\iota+1}(B,y')/\sigma_z\right)}{\exp\left(\mathcal{F}^d_{\iota+1}(B,y')/\sigma_z\right) + \sum_j \exp\left(\mathcal{F}^r_{\iota+1}(B^j,y')/\sigma_z\right)} ,$$
$$\Pr(B' = B^j|B,y') = \frac{\exp\left(\mathcal{F}^r_{\iota+1}(B^j,y')/\sigma_z\right)}{\exp\left(\mathcal{F}^d_{\iota+1}(B,y')/\sigma_z\right) + \sum_j \exp\left(\mathcal{F}^r_{\iota+1}(B^j,y')/\sigma_z\right)} .$$

(d) Determine the default policy $\mathcal{D}$ and expected payoffs k^r and k^d w.r.t. z using the probabilities computed in (c):

$$\begin{aligned}
\mathcal{D}_{\iota+1}(B,y') =& \Pr(d=1|B,y') ,\\
k^r_{\iota+1}(B,y') =& \sum_j \Pr(B^j|B,y')\left[\widetilde{\delta} + (1-\delta)\mathcal{Q}^r_\iota(B^j,y')\right] ,\\
k^d_{\iota+1}(B,y') =& \vartheta\omega \left\{ \begin{array}{c} (1-\mathcal{D}_\iota(\omega B, P(\omega B,y'),y')) \times \left(\widetilde{\delta} + (1-\delta)\mathcal{Q}^r_\iota(\omega B,y')\right) \\ +\mathcal{D}_\iota(\omega B,y')\mathcal{Q}^d_\iota(\omega B,y') \end{array} \right\} + (1-\vartheta)\mathcal{Q}^d_\iota(B,y') ,
\end{aligned}$$

where the bond prices under reentry $\mathcal{Q}^r_\iota(\omega B,y')$ and $\mathcal{Q}^d_\iota(\omega B,y')$ are interpolated linearly, since ωB is not necessarily part of the debt grid $\mathfrak{B}$.

(e) Update expected value functions $\mathcal{F}$, default probability λ and pecuniary payoffs m w.r.t. the persistent exogenous state:

$$\begin{aligned}
&F_{\iota+1}(B,y) = \Pi\mathcal{F}_{\iota+1}(B,y') , \quad F^d_{\iota+1}(B,y) = \Pi\mathcal{F}^d_{\iota+1}(B,y'),\\
&m^r_{\iota+1}(B,y) = \Pi k^r_{\iota+1}(B,y') , \quad m^d_{\iota+1}(B,y) = \Pi k^d_{\iota+1}(B,y'),\\
&\lambda_{\iota+1}(B,y) = \Pi\mathcal{D}_{\iota+1}(B,y').
\end{aligned}$$

(f) Compute haircuts κ, haircut-weighted collateral Θ, and adjusted bond holdings for both investor types in the repayment case, and combine these elements to obtain the non-pecuniary part of the payoff, Λ:

$$
\begin{aligned}
\kappa^r_{\iota+1}(B,y) =& \kappa(\lambda_{\iota+1}) \,, \\
\Theta^r_{\iota+1}(B,y) =& (1-\kappa^r_{\iota+1}(B,y)) \times m^r_{\iota+1}(B,y') \times B \,, \\
\Theta^d_{\iota+1}(B,y) =& (1-\overline{\kappa}) \times m^d_{\iota+1}(B,y') \times B \,, \\
\Theta^r_{H,\iota+1}(B,y) =& \Theta^r_{\iota+1}(B,y) - \frac{1}{2}\zeta_2 \,, \\
\Theta^d_{H,\iota+1}(B,y) =& \Theta^d_{\iota+1}(B,y) - \frac{1}{2}\zeta_2 \,, \\
\Theta^r_{L,\iota+1}(B,y) =& \Theta^r_{\iota+1}(B,y) + \frac{1}{2}\zeta_2 \,, \\
\Theta^d_{L,\iota+1}(B,y) =& \Theta^d_{\iota+1}(B,y) + \frac{1}{2}\zeta_2 \,, \\
\Lambda^r_{\iota+1}(B,y) =& (1-\kappa^r_{\iota+1}(B,y))\Big(\frac{1}{2}\big(\frac{1}{2}u'\big(\Theta^r_{L,\iota+1}(B,y')\big) - \frac{1}{2}u'\big(\Theta^r_{\iota+1}(B,y')\big)\big) \\
& + \frac{1}{2}\big(\frac{1}{2}u'\big(\Theta^r_{H,\iota+1}(B,y')\big) - \frac{1}{2}u'\big(\Theta^r_{\iota+1}(B,y')\big)\big)\Big) \\
\Lambda^d_{\iota+1}(B,y) =& (1-\overline{\kappa})\Big(\frac{1}{2}\big(\frac{1}{2}u'\big(\Theta^d_{L,\iota+1}(B,y')\big) - \frac{1}{2}u'\big(\Theta^d_{\iota+1}(B,y')\big)\big) \,, \\
& + \frac{1}{2}\big(\frac{1}{2}u'\big(\Theta^d_{H,\iota+1}(B,y')\big) - \frac{1}{2}u'\big(\Theta^d_{\iota+1}(B,y')\big)\big)\Big) \,.
\end{aligned}
$$

(g) Update bond prices in repayment and default :

$$
\begin{aligned}
\mathcal{Q}^r_{\iota+1}(B,y) &= m^r_{\iota+1}(B,y)\big(1+\Lambda^r_{\iota+1}(B,y)\big)/(1+r^{rf}), \\
\mathcal{Q}^d_{\iota+1}(B,y) &= m^d_{\iota+1}(B,y)\big(1+\Lambda^d_{\iota+1}(B,y)\big)/(1+r^{rf}) \,.
\end{aligned}
$$

(h) Apply the cap on bond prices.

2. Compute convergence criteria:

$$
\begin{aligned}
\Delta\mathcal{F}_{\iota+1} &= \max\left\{ \Big|\Big|\mathcal{F}_{\iota+1} - \mathcal{F}_{\iota}\Big|\Big|_\infty, \Big|\Big|\mathcal{F}^d_{\iota+1} - \mathcal{F}^d_{\iota}\Big|\Big|_\infty \right\} \\
\Delta\mathcal{Q}_{\iota+1} &= \max\left\{ \Big|\Big|\mathcal{Q}^r_{\iota+1} - \mathcal{Q}^r_{\iota}\Big|\Big|_\infty, \Big|\Big|\mathcal{Q}^d_{\iota+1} - \mathcal{Q}^d_{\iota}\Big|\Big|_\infty \right\}
\end{aligned}
$$

3. If $\Delta\mathcal{Q}_{\iota+1} < \varepsilon \wedge \Delta\mathcal{W}_{\iota+1} < \varepsilon$, stop. Otherwise, set $\iota = \iota + 1$ and go to step 1.

Table B.2 summarises the parameters governing the numerical approximation. The results do not visibly change when the grid sizes are increased further. The standard deviation of the taste shock $\sigma_z = 0.5$ is chosen to ensure stable convergence for all parameter combinations, which is typically achieved in less then 1000 iterations.

Following Chatterjee and Eyigungor (2015), we impose a maximum default probability to rule out extreme debt dilution when the economy is close to default. The maximum default probability is set to 0.75. To rule out implausibly high valuations of collateral services for small amounts of outstanding debt, we also impose a minimum bond spread (or, alternatively, maximum convenience yield). The minimum spread of a five-year German government bond over the five-year interest-rate-swap rate observed in our sample, which was -105 bps, serves as a lower bound on bond spreads in our calibration. Like the maximum default probability, this constraint does not bind during model simulation and all of our numerical results are robust to changes to either constraint.

Table B.2: Parameters of the computational algorithm

Number of grid points for income	$n_y = 101$
Size of income grid	$y \in [-3\sigma_y, +3\sigma_y]$
Number of grid points for debt	$n_B = 301$
Size of debt grid	$B \in [0, 6]$
Taste shock parameter	$\sigma_z = 0.5$
Maximum default probability	$\overline{\lambda} = 0.75$
Minimum bond spread	$\underline{s} = -105$ bps

C Additional quantitative results

C.1 Model fit with inelastic collateral valuation

In this section, we demonstrate that imposing a debt-inelastic collateral valuation component, obtained by setting $\zeta_2 = 0$, yields a substantially worse fit with the data than our baseline calibration. Specifically, the volatility of convenience yield falls well short of its data moment. Its increase (in absolute terms) during crisis episodes is also much less pronounced. Compared to the effect of debt supply on convenience yield in the baseline calibration (-35, see Table 4), the coefficient on debt supply in (22) increases to +3 in the case of constant collateral valuation, which is also not consistent with the data.

Table C.1: Model fit with $\zeta_2 = 0$

Variable	Full sample		Safe episodes		Crisis episodes		Volatility	
	Data	Model	Data	Model	Data	Model	Data	Model
s_t	48	48	-9	-10	139	176	114	172
cds_t	78	76	15	18	192	207	113	175
cy_t	-30	-30	-23	-28	-54	-31	29	4
$\log(Q_t B_t / y_t)$	1.51	1.49	1.46	1.47	1.57	1.59	0.058	0.084

Note: Spreads are annualised and given in basis points, y_t refers to quarterly real GDP. Crisis episodes are all periods with a positive government bond spread. Targeted moments are shown in colour. All model-implied statistics are based on simulations of 50,000 periods, excluding 5,000 burn-in periods. We also exclude all periods in which the government is in financial autarky as well as 40 quarters after reentering financial markets following an exclusion period.

C.2 Model extensions

This sub-section provides additional details on both of the model extensions presented in Section 6. For each extension, we briefly outline how the functional forms are modified, how we recalibrate free model parameters, and how the model fit changes. We subsequently show that the comparative statics with respect to haircut elasticities remain intact in both extensions.

Market illiquidity. To take market illiquidity into account, we specify the period utility function of high-valuation and low-valuation investors as follows:

$$u_L(\theta_t) = -\frac{\zeta_1}{\zeta_2} \exp\{-\zeta_2 \theta_t\} ,$$
$$u_H(\theta_t) = -\frac{\zeta_1}{\zeta_2} \exp\{-\zeta_2 (\theta_t - \zeta_3 - \zeta_5 \mathbf{1}\{\log(y_t) < 0\})\} .$$

It follows that the wedge between bond and CDS prices is given by

$$\Lambda_t = (1 - \kappa_t) \zeta_1 \exp\{-\zeta_2 (1 - \kappa_t) m_t B_{t-1}\}.$$

The optimal bond holdings of high-valuation and low-valuation investor types are given by

$$\theta_{L,t} = (1-\kappa_t)m_t B_{t-1} - \frac{1}{2}(\zeta_3 + \zeta_5 \mathbf{1}\{\log(y_t) < 0\}) \,,$$
$$\theta_{H,t} = (1-\kappa_t)m_t B_{t-1} + \frac{1}{2}(\zeta_3 + \zeta_5 \mathbf{1}\{\log(y_t) < 0\}) \,.$$

We can use parameter ζ_3 to tune the trading motif between high-valuation and low-valuation investors. Parameter ζ_5 allows us to generate the empirically observed uptake in bid-ask spreads during sovereign debt crises in a parsimonious way. For a micro-founded model that is capable of explaining this feature, we refer to Chaumont (2018) and Passadore and Xu (2022). It is natural to target the averages and volatilities of bid-ask spreads when setting $\zeta_3 = 1e-05$ and $\zeta_5 = 0.001$. As in our empirical analysis (Table 4), the data moment for bid-ask spreads is computed using the on-the-run five-year-bond. The model fit is shown in Table C.2.

Table C.2: Model fit with market illiquidity

Variable	Sample mean		Safe episodes		Crisis episodes		Volatility	
	Data	Model	Data	Model	Data	Model	Data	Model
s_t	48	45	-9	-6	139	164	114	144
cds_t	78	76	15	16	192	215	113	176
cy_t	-30	-31	-23	-22	-54	-51	29	33
$\log(Q_t B_t / y_t)$	1.51	1.51	1.46	1.47	1.57	1.59	0.058	0.084
ba_t	11	12	3	4	29	28	16	39

Note: Spreads are annualised and given in basis points, y_t refers to quarterly real GDP. Crisis episodes are all periods with a positive government bond spread. Targeted moments are shown in colour. All model-implied statistics are based on simulations of 50,000 periods, excluding 5,000 burn-in periods. We also exclude all periods in which the government is in financial autarky as well as 40 quarters after reentering financial markets following an exclusion period.

Time-varying collateral demand. In a second extension, we modify the collateral demand of investors as follows:

$$u(\theta_t) = -\frac{\zeta_1 + \zeta_4 \mathbf{1}\{\log(y_t) < 0\}}{\zeta_2} \exp\{-\zeta_2 \theta_t\} \,.$$

To obtain a higher collateral demand in times of crisis, we set $\zeta_4 = 0.5$ and recalibrate the weighting parameter to $\zeta_1 = 0.03$, which ensures an average convenience yield of -30 bps. Notably, this extension also affects the elasticity parameter in the collateral valuation function, which we reduce to $\zeta_2 = 1.3$ in order to match the time series volatility of convenience yield. Although this value is slightly smaller than in the baseline calibration, it still implies a quite substantial curvature in the collateral valuation function. Furthermore, we recalibrate $\beta = 0.971$ and $d_0 = 20.3125$ to match the average CDS spreads and debt-to-GDP ratio. The model fit is summarised in Table C.3 and is overall very similar to the baseline calibration.

Table C.3: Model fit with $\zeta_4 = 0.5$

Variable	Full sample		Safe episodes		Crisis episodes		Volatility	
	Data	Model	Data	Model	Data	Model	Data	Model
s_t	48	49	-9	1	139	155	114	142
cds_t	78	78	15	17	192	211	113	176
cy_t	-30	-29	-23	-16	-54	-56	29	35
$\log(Q_t B_t / y_t)$	1.51	1.47	1.46	1.43	1.57	1.55	0.058	0.083

Note: Spreads are annualised and given in basis points, y_t refers to quarterly real GDP. Crisis episodes are all periods with a positive government bond spread. Targeted moments are shown in colour. All model-implied statistics are based on simulations of 50,000 periods, excluding 5,000 burn-in periods. We also exclude all periods in which the government is in financial autarky as well as 40 quarters after reentering financial markets following an exclusion period.

C.3 Robustness: No haircut cap

In this section, we investigate the role of the haircut cap $\overline{\kappa}$. We hold the CARA parameter in the collateral valuation function at its baseline value $\zeta_2 = 1.5$ and recalibrate $\zeta_1 = 0.425$. This slight increase over the baseline value of $\zeta_1 = 0.4$ is necessary since average haircuts (and, thus, average convenience yield) is smaller without the cap on haircuts. To match CDS spreads and the debt-to-GDP ratio, we lower the discount factor to $\beta = 0.968$ and increase the default cost parameter to $d_0 = 22.1875$. The model fit is shown in the upper panel of Table C.4. While the full-sample averages are very similar to the baseline calibration with a haircut cap it is noteworthy that the volatility of convenience yield and its decline during crisis episodes fall short of the data moment. In the lower panel of Table C.4, we demonstrate that excluding collateral scarcity effects is also not consistent with the time series variation of convenience yield under the $\overline{\kappa} = 1$ restriction.

In Table C.5, we compare the effects of varying haircut elasticity μ for the debt-elastic case and the debt-inelastic case. The effect of μ on debt issuance and default risk is even larger than in the baseline parameterisation with a haircut cap. For the debt-elastic case, the average CDS spread declines by more than 30 bps when the haircut elasticity is relaxed. Likewise, the default rate declines by almost half a percentage point. The reason behind this is that the haircut cap makes the haircut schedule debt-inelastic for high values of default risk, irrespective of the elasticity parameter μ. Similarly, the right-hand panel shows that tightening haircuts has somewhat stronger effects than in the baseline for the debt-inelastic case as well. Overall, the comparative static exercise yields qualitatively identical results to the baseline, suggesting that haircut caps are not driving our analysis in any crucial way.

Table C.4: Model fit with $\overline{\kappa} = 1$

With scarcity	Sample mean		Safe episodes		Crisis episodes		Volatility	
	Data	Model	Data	Model	Data	Model	Data	Model
s_t	48	51	-9	-6	139	182	114	162
cds_t	78	78	15	17	192	219	113	179
cy_t	-30	-27	-23	-23	-54	-37	29	19
$\log(Q_t B_t / y_t)$	1.51	1.50	1.46	1.47	1.57	1.57	0.058	0.076
Without scarcity	Sample mean		Safe episodes		Crisis episodes		Volatility	
	Data	Model	Data	Model	Data	Model	Data	Model
s_t	48	51	-9	-11	139	190	114	175
cds_t	78	75	15	18	192	219	113	176
cy_t	-30	-28	-23	-29	-54	-29	29	2
$\log(Q_t B_t / y_t)$	1.51	1.49	1.46	1.47	1.57	1.55	0.058	0.069

Note: Spreads are annualised and given in basis points, y_t refers to quarterly real GDP. Crisis episodes are all periods with a positive government bond spread. Targeted moments are shown in colour. All model-implied statistics are based on simulations of 50,000 periods, excluding 5,000 burn-in periods. We also exclude all periods in which the government is in financial autarky as well as 40 quarters after reentering financial markets following an exclusion period.

Table C.5: Haircut and collateral valuation components with $\overline{\kappa} = 1$

	With scarcity			*Without scarcity*		
	$\mu = 0.1$	$\mu = 0.4$	$\mu = 0.7$	$\mu = 0.1$	$\mu = 0.4$	$\mu = 0.7$
$\text{ave}(s_t)$	68	51	39	42	47	48
$\text{ave}(cds_t)$	99	78	65	71	75	77
Default rate (%)	1.64	1.38	1.09	1.23	1.29	1.29
$\text{ave}(B_t / y_t)$	4.10	4.00	3.96	3.95	3.96	3.96
$\text{ave}(q_t B_t / y_t)$	4.56	4.48	4.47	4.45	4.45	4.45

Note: Effects of varying the haircut parameter μ. A low μ corresponds to a highly elastic haircut schedule (see also Figure 2). Spreads are annualised and given in basis points, y_t refers to quarterly real GDP. All model-implied statistics are based on simulations of 50,000 periods, excluding 5,000 burn-in periods. We also exclude all periods in which the government is in financial autarky as well as 40 quarters after reentering financial markets following an exclusion period.